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# Efficiency Analysis of Queueing Model Validation by Use of Trace-Driven Simulation

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Validation of queueing models is the main practical task. *The Trace-driven simulation* approach is often used for this purpose (Kleijnen et al. 1999, Andronov 2000, 2001). This approach can be described in the following way. We suppose that a real queueing system of interest has an observed input $A_1, A_2, ..., A_n$ and an observed output or replicate $W_1, W_2, ..., W_n$. For example, $A_1, A_2, ..., A_n$ form a sequence of arrival times of customers into a queueing system, $W_1, W_2, ..., W_n$ form a sequence of sojourn times of the customers in the system or the output times from the system. The output series are evaluated through a performance measure (response) $X$, the average sojourn time of the customers, for example.

We have a theoretical queueing model as well. We should verify its adequacy to the real system, it is the so-called *null-hypothesis*. For this purpose we use the same *trace* $\mathbf{A} = (A_1, A_2, ..., A_n)$ and produce the simulation $R$ times. As a result we get $R$ simulated outputs (replicates) $\tilde{W}_1^{(r)}, \tilde{W}_2^{(r)}, ..., \tilde{W}_n^{(r)}$, where $r$ is a run number of the simulation, $r = 1, 2, ..., R$. Let $\tilde{X}^{(1)}, \tilde{X}^{(2)}, ..., \tilde{X}^{(R)}$ be a corresponding sequence of the performances (responses).

To validate the theoretical model (to test the null-hypothesis), the real and simulated performances $X$ and $\tilde{X}^{(1)}, \tilde{X}^{(2)}, ..., \tilde{X}^{(R)}$ should be compared statistically. Some solutions of this problem were given in (Kleijnen et al. 1999). We consider another approach. The simplest way supposes a sorting of the simulated performances $\tilde{X}^{(1)}, \tilde{X}^{(2)}, ..., \tilde{X}^{(R)}$. It gives the order statistics $\tilde{X}_{(1)}, \tilde{X}_{(2)}, ..., \tilde{X}_{(R)}$ and the estimated $\alpha$-quantile of the distribution $\tilde{X}_{(\lfloor R\alpha \rfloor)}$. This procedure gives, for example, a two-sided $(1 - \alpha)$-confidence interval for the original value $X$, ranging from the lower estimated $\alpha/2$-quintile to the upper $(1 - \alpha/2)$-quintile. If $X$ value falls outside this interval then we reject the theoretical model (the null-hypothesis). In addition, the significance level of this testing is equal to $\alpha$.

Our aim is to investigate the efficiency of the described approach for a quite general case of queueing system. We shall consider these systems as a composite two-component embedded Semi-Markov process. Here the input of the queueing system $\mathbf{A}$ corresponds to a Semi-Markov process $A(t)$. The latter is described by a discrete set $\Omega_A$ of states, matrix of one-step transition probabilities $\mathbf{P} = (P_{i,j})$ and the distribution function $F_i(t)$ of sojourn time in each state $i \in \Omega_A$. Obviously,

$$\sum_{j \in \Omega_A} P_{i,j} = 1, \quad F_i(\infty) = 1, \quad \forall i \in \Omega_A.$$

Remind (Ross, 1992, p.86) that "Semi-Markov process records the state of the process at each time point $t$". So we have these states as the considered input $\mathbf{A} = (A_1, A_2, ..., A_n)$.

Further we suppose that the inner structure of our queueing system is described by a stochastic process $S(t)$ with a discrete set of states $\Omega_S$. If the state $J \in \Omega_A$ of the Semi-Markov process $A(t)$ takes place then the process $S(t)$ behaves as an independent Semi-Markov process with the matrix $\mathbf{P}^{S,J}$ of one-step transition probabilities and the distribution function $F_i^{S,J}(\cdot)$ of sojourn time in the state $i \in \Omega_S$. We shall name jumps (transitions) of the process $A(t)$ as jumps of the first type and supplemental jumps of the process $S(t)$ only as the jumps of the second type. Note that the probability to have a jump of the process $\mathbf{A}(t)$ at a time $t$ does not depend on the state of $S(t)$.

Let $t^*$ be a time moment when a jump of the first type takes place, $J_- = A(t^*-)$ and $J_+ = A(t^*+)$ be the states of the process $A(t)$ immediately before and after this time moment. Let $J_-^S = S(t^*-)$ and $J_+^S = S(t^*+)$ be the same for the process $S(t)$. If $J_+$ is entered, the next state $J_+^S$ of the process $S(t)$ is chosen according to the transition probabilities $q_{J_-^S,J_+^S}^{J_-,J_+}$. Then given that the state chosen is $J_+^S$, the time until transition of the process $S(t)$ has the distribution function $F_{J_+^S}^{S,J_+}(\cdot)$ as usually for $S(t)$.

Now we consider the sequence of time moments $t_1, t_2, ...$ when all jumps occur. Every one of such moments produces a random variable $\tilde{W}$, so $\tilde{W}_1, \tilde{W}_2, ...$ form the output of our system. To describe the corresponding stochastic mechanism, we introduce the following notations for the jump of the first type at the time moment $t^*$: $\tau(t^*)$ - length of the interval between the previous and current jumps of the first type; $U(t^*)$ - time till $t^*$ since the last jump of the processes $S(t)$, so-called *age* of the current state at $t^*$. Then the probabilities $Pr_1(J_-, J_+, J_-^S, J_+^S, \tau(t^*), U(t^*))$ to produce the random variables $\tilde{W}$ are functions of $J_-, J_+, J_-^S, J_+^S, \tau(t^*)$ and $U(t^*)$. For the jumps of the second type this probability is denoted by $Pr_2(J, J_-^S, J_+^S, U(t^*))$. If the corresponding random variable $\tilde{W}$ will be produced, then its distribution function is given by the formulas

$$G_1^S(x; J_-, J_+, J_-^S, J_+^S, \tau(t^*), U(t^*)) = P\{\tilde{W} \le x / J_-, J_+, J_-^S, J_+^S, \tau(t^*), U(t^*)\},$$

$$G_2^S(x; J, J_-^S, J_+^S, U(t^*)) = P\{\tilde{W} \le x / J, J_-^S, J_+^S, U(t^*)\}.$$

The sequence $\{\tilde{W}_i\}$ is an output from the simulated system with the inner structure, that is described by the process $S(t)$. Let the inner structure of the real system be described by the random process $B(t)$. We suppose that the process $B(t)$ is similar to the process $S(t)$ but may have some different parameters. Let $W_1, W_2, ...$ denote the output for the real process $B(t)$ if the same input $\mathbf{A}$ takes place. We must compare two sequences $W_i$ and $\tilde{W}_i$ statistically. For this purpose we use some performance measure $X = \xi(W_1, W_2, ...)$ and $\tilde{X} = \xi(\tilde{W}_1, \tilde{W}_2, ...)$.

Let us calculate a conditional distribution function $R_S(x/A)$ of $\tilde{X}$ under the condition that the input (trace) $\mathbf{A}$ is fixed, for example, by simulation:

$$R_S(x/\mathbf{A}) = P\{\tilde{X} \le x/\mathbf{A}\}.$$

Let $R_S^{-1}(\gamma/\mathbf{A})$ be the corresponding $\gamma$-quantile: $R_S(R_S^{-1}(\gamma/\mathbf{A})/\mathbf{A}) = \gamma$.

Now we are able to test the null hypothesis $H_0$ that random processes $B(t)$ and $S(t)$ have the same probabilistic structure against the alternative hypothesis $H_1$ that they are different. Let us suppose that if the alternative hypothesis $H_1$ is true then the performance measure $X$ is stochastically greater than $\tilde{X}$. In this case we reject the null-hypothesis for the significance level $\alpha$ if the fixed value $X$ exceeds $R_S^{-1}(1-\alpha/\mathbf{A})$. In other words the decision interval for the null-hypothesis $H_0$ is $(0, R_S^{-1}(1-\alpha/\mathbf{A}))$. What is *the power function* for this test? We consider this problem in our paper.

## REFERENCES


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