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# No-Waiting Stations with Spatial Arrival Processes and Customer Motion

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**Abstract**—More realistic models of wireless mobile communication networks have to take into account spatial occurrence as well as movement of customers. In several configurations of practical interest queueing of customers is not provided. For example, in most cellular mobile communication systems, calls are accepted immediately as long as total capacity tolerates it, and are lost, if the system is overloaded. That way the analyst, when using the framework of queueing theory, inevitably is led to the investigation of infinite server models or finite capacity loss systems without waiting room. In this paper, starting from the above motivation, stations with infinitely many servers, general service time distribution, and Coxian or Markov-additive arrival processes are investigated. Extensions include spatial variants of the arrival process and the possibility of customer movements in space. Transient as well as steady state distributions are obtained for the number of customers observable in certain Borel subsets of some prechosen area of interest. In contrast to similar results in previous work, where also an approximate solution for the $BMAP/G/c/0$ – model was presented[1] $[1, 3, 5]$, the basic stochastic differential equations for the $BMAP/G/\infty$ – model are obtained in neat form as component expressions of the Kolmogorov forward equations.
This paper has been elaborated at the beginning of the year 2001, and has been the subject of discussions — partly via e-mail — with my late esteemed friend, Professor Vladimir V. Kalashnikov, until the 19th of March, 2001.

## 1. THE BMAP/G/$\infty$ – MODEL

Let $\{(N_t, J_t) : t \geq 0\}$ denote a batch Markovian arrival process (BMAP) as defined in [7] and [6]. $N_t$ describes the additive component, $J_t$ the phase variable (we assume finiteness of its state space $E$). The BMAP is characterized by the sequence $\Delta = \{D_0, D_1, D_2, \ldots\}$ of rate matrices; $D = \sum_{n=0}^{\infty} D_n$ is the generator of the phase process.

### 1.1 Transient and Steady State Analysis

For $u \leq t$, let $N_{u,t}$ be the number of customers arrived until time $u$ and still being in service at time $t$. In particular, $N_u = N_{u,u}$. Assume that $N_0 = 0$, and set $Q_{r;ik}(u,t) = \mathbb{P}_i(N_{u,t}(S) = r, J_u = k)$. With $F(\cdot)$ the service time distribution, the process $\mathfrak{S}(F,t) = (N_{u,t}, J_u)$ for fixed $t$, $0 \leq u \leq t$, is non-homogeneous Markov. A key step in the analysis is the formulation of Kolmogorov's forward differential equation and its (in general iterative) solution method. For that, the *residental rate matrices* $R_\nu(n;\, u,t)$ $(\nu, n \in \mathbb{N}_0)$ are defined with means of the $D_n$ and $F$. The entry $(R_\nu(n;\, u,t))_{ij}$ denotes the arrival rate of $\nu$ t-resident customers due to a single batch arrival of size $n$ together with a phase transition from $i$ to $j$ at time epoch

[1] The entry "0"means "zero waiting places". This Kendall-notation is not universally valid; e. g., in Anglo-American literature the notation $BMAP/G/c/c$ is used in this case.

$u \leq t$ (assuming, that the process starts with an empty system)[2]. The generator $\mathfrak{G}(u)$ of $\mathfrak{S}(F, t)$ is given then through $G_{(m,i)(n,j)}(u) = R_{n-m;i,j}(u)$ for $(m, i) \neq (n, j)$. Setting $Q_{m,n;i,j}(v, u; t) = \mathbb{P}_i(N_{u,t} = n, J_u = j \mid N_{v,t} = m))$, $\mathfrak{Q}(u, v) = (\mathcal{Q}_{m,n}(v, u))_{i,j \in E,\, m,n \in \mathbb{N}_0}$, and $\mathfrak{G}(u) = (\mathcal{G}_{m,n}(u))_{i,j \in E,\, m,n \in \mathbb{N}_0}$, one obtains

$$\frac{\partial \mathfrak{Q}(u, v)}{\partial u} = \mathfrak{Q}(v, u) \cdot \mathfrak{G}(u) \ . \tag{1}$$

The iterative solution method for (1) is demonstrated in the paper. An analytic solution in convolutional exponential form is known [2] for the special case of mutually commuting rate matrices $D_n$. The matrix $\mathcal{Q}_{0,r}(0, t)_{i,j \in E} =: (Q_r(t))_{i,j \in E}$ describes phase dependent transient state probabilities. Equilibrium state probabilities can be found as the limits $Q_{r;j} = \lim_{t \to \infty} (Q_r(t))_{i,j}$, independently of $i$.

1.2 Variants (Spatial Arrival Process, Moving Customers)

The use of spatial BMAPs as introduced in [4] leads to corresponding results for the transient as well as steady state distribution of the number of customers in some measurable subset $S$ of an area $\mathcal{R}$ in space ($\mathcal{R}$ denoting some subset of a Polish space, e. g., the $\mathbb{R}^2$). The meaning of $\mathcal{R}$ depends on the actual modelling problem. The movement of customers in space is modelled by a collection of group operations $\{\Upsilon_s\}_{-\infty < s < \infty}$ (satisfying $\Upsilon_{s+t} = \Upsilon_s \Upsilon_t$), for which $\Upsilon_s(\mathbf{x}) = \mathbf{x}(s)$ denotes the position at time $s$ of a customer who arrived at time zero at position $\mathbf{x}$. We extend previous results by describing the joint distribution for any collection $S_1, S_2, \ldots, S_n$ of disjoint Borel subsets.

## 2. THE COX/G/∞ – MODEL

If no simultaneous occurrence of arrival and phase transition is allowed, the analysis can be extended to stations with spatial Cox arrival process. Here the arrival intensities are controlled by some arbitrary random phase process $\{J_t\}_{t \geq 0}$ with cadlag trajectories and (not necessarily finite) denumerable state space $E$. Considering an arbitrary measurable subset $S$ of the basic spatial area $\mathcal{R}$, the probability matrix $Q_r(S; t)$ of observing $r$ customers at time $t$ in $S$ can be expressed for the infinite server model as

$$Q_r(S; t) = \frac{1}{r!}\, \mathbb{E}\left[\Theta^r(S; t) \exp(-\Theta(S; t))\right] \ , \quad t \geq 0 \,,$$

where $\Theta(S; t) = \int_0^t \xi_{J_{t-u}}(\Upsilon_{-u}[S]) \overline{F}_{J_{t-u}}(u)\, du$, $\xi_{J_{t-u}}(\Upsilon_{-u}[S])$ is the arrival intensity to subset $\Upsilon_{-u}[S]$ at time $t - u$, and $\overline{F}_{J_{t-u}}(u)$ the complementary service time distribution function during phase $J_{t-u}$ [3]. These facts are also settled in terms of generating functions. As already shown in [3], there is an immediate extension to joint distributions over a collection of disjoint subsets $S_1, S_2, \ldots, S_n$. Further, for a time-homogeneous phase process with finite state space, we obtain limiting values for the expectations $\mathbb{E}[N_t(S_i)]$ and $\mathbb{E}[N_t(S_i), N_t(S_j)]$ $(1 \leq i, j \leq n)$ as $t \to \infty$. These results are extended in our paper to the case of a compound spatial Cox arrival process.

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