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Queueing Network Simulation Based on Quasi-weak Regeneration[1]

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We discuss the possibilities of the weak regeneration in the steady-state analysis of networks and offer a new method for regeneration point detection – quasi-weak regeneration.

First, we recall some main notions related to regenerative approach, [1]. We consider a process $\{Z(t), t \geq 0\}$ with the sequence of its *regeneration points (r.p.'s)* $\beta = (\beta_1, \beta_2, \ldots)$. Under strong regeneration, the trajectory parts between r.p.'s– *regeneration cycles* - are i.i.d. with the (i.i.d.) cycle lengths ($\alpha_k = \beta_{k+1} - \beta_k$). Weak regeneration allows dependence between cycles while cycle lengths stay i.i.d. (In fact, in the network context we can restrict ourselves to the m-dependent case, [3]). Under known stability assumptions, the limit distribution of regenerative process exists, $Z(t) \Rightarrow Z$, [1]. Assuming the time is discrete, we denote

$$Y_i = \sum_{k=\beta_i}^{\beta_{i+1}-1} f(Z_k),\ V_i = Y_i - \mathbb{E}f(Z)\alpha_i,\ i \geq 1, \tag{1}$$

(so $\{V_i\}$ are i.i.d with $\mathbb{E}V_1 = 0$), and let $\mathbb{E}f(Z)$ be a steady-state characteristic of the process.

The approach based on strong regeneration (and classic CLT) results in the following asymptotic $100(1-2\gamma)\%$ confidence interval for $\mathbb{E}f(Z)$:

$$I = \left[\, \hat{r}_n - \frac{z_\gamma S_n}{\sqrt{n}\bar{\alpha}_n};\ \hat{r}_n + \frac{z_\gamma S_n}{\sqrt{n}\bar{\alpha}_n} \,\right], \tag{2}$$

where $\bar{\alpha}_n$ and $\bar{Y}_n$ are the sample means of $\{\alpha_j\}$ and $\{Y_i\}$ respectively, $\hat{r}_n = \bar{Y}_n/\bar{\alpha}_n$, z_γ is the correspondent quantile of the (standard) normal distribution and S_n^2 is the standard (strongly consistent and unbiased) estimate of

$$Var(V_1) = Var(Y_1) + Var(\alpha_1) - 2\mathbb{E}f(Z)cov(Y_1, \alpha_1).$$

We note that different sequences of strong r.p.'s lead to the same asymptotic confidence intervals [3], but they can result in different simulation time required to construct the confidence interval.

Unlike in the classic case, weak regeneration leads to the appearance of $cov(V_i, V_j)$ in the expression $Var(\sum V_i)$, where (due to independence of $\{\alpha_1\}$ and $\{\alpha_2\}$ and also of $\{\alpha_1\}$ and Y_2)

$$cov(V_1, V_2) = cov(Y_1, Y_2) - \mathbb{E}f(Z)cov(Y_1, \alpha_2). \tag{3}$$

Then the CLT (for m-dependent variables, [3]) gives the following confidence interval for $\mathbb{E}f(Z)$:

$$I = \left[\hat{r}_n - \frac{z_\gamma\sqrt{S_n^2 + 2(t_1(n) - \hat{r}_n t_2(n))}}{\sqrt{n}\bar{\alpha}_n};\ \hat{r}_n + \frac{z_\gamma\sqrt{S_n^2 + 2(t_1(n) - \hat{r}_n t_2(n))}}{\sqrt{n}\bar{\alpha}_n}\right], \tag{4}$$

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where $t_1(n)$, $t_2(n)$ are standard estimates of $cov(Y_1, Y_2)$, and $cov(\alpha_2, Y_1)$, respectively, [3].

Let us consider the Jackson-type network with $N < \infty$ nodes (stations) numbered $1, \dots, N$, where node i has $m_i \geq 1$ parallel channels of service (servers). We suppose FIFS as the service discipline, which is called FIFO in the single-server case. Input for each node consists of internal and external inputs.

Let $\tau^i = \{\tau_n^i = t_{n+1} - t_n\}_{n \geq 1}$ be the external input to the node with the i.i.d interarrival times $\{\tau_n^i\}_{n \geq 1}$; $\{s_n^i\}_{n \geq 1}$ – i.i.d service times in node i, $i = 1, \dots, N$; and $P = \|P_{ij}\|_{i \geq 0, j \geq 0}$ be transient matrix (i.e., after node i the customer goes to node j with the probability P_{ij}). We consider node 0, sush that the arrival of a customer to this node means the departure from the network $\left(\text{with the probability } P_{i0} = 1 - \sum_{k=1}^{N} P_{ik} \right)$. It is known ([2]) that the condition

$$\rho_i = \frac{Es_1^{(i)}}{E\tau_1^{(i)}} < m_i, \; i = 1, \dots, N. \tag{5}$$

implies stability of the network.

Below we describe detection of regeneration points, the so-called quasi-weak (q-weak) regeneration points.

By the *dependent* customers we mean collided (clashed) ones, those standing in one queue on some channel of any node simultaneously. Furthermore, if n_1 and n_2 $(n_1 < n_2)$ are numbers of collided customers, then we call all the customers with numbers $k: \; n_1 \leq k \leq n_2$ dependent. The i-th set of dependent customers is called **dependence group** and denoted by B_i. Thus, if customers n_1-th,n_2-th $\in B_i$, they are dependent. Let $[a_i, b_i]$ be the bounds of the dependence group B_i, where a_i, b_i – positive integers (as customers' numbers), and $B = \{B_i\}$ – the set of dependence groups.

The set of dependence groups changes if a customer clash occurs (some customer arrives to a nonempty channel and collides with the customer ahead). By definition, we can represent the set of dependence groups as intervals $[a_1, b_1], [a_2, b_2], \dots$. Hence, if customers n_1-th and n_2-th collide at some moment, we intersect the intervals $[n_1, n_2]$ and $\{[a_i, b_i]\}_{i \geq 1}$ to obtain a new set of dependence groups.

Now we introduce the definition of the **release group**. The release group consists of consequent numbers of departed (released) customers. Let us denote the i-th release group by R_i. Hence, if n_1,$n_2 \in R_i$ $(n_1 < n_2)$, then all the customers with numbers $k: \; n_1 \leq k \leq n_2$, belong to the group R_i and it means they have left the network. Let $[c_i, d_i]$ be the bounds of release group R_i, where c_i, d_i are positive integers (as customers' numbers), and $R = \{R_i\}$ is the set of release groups.

There are four variants of changing the set of release group at the departure moment of the n-th customer:

1. Suppose that for the group R_i the bound $d_i = n - 1$, then the customer joins the group R_i and we obtain a new bound $d_i = n$;
2. Assume for the group R_i the bound $c_i = n + 1$, then the customer joins the group R_i and we obtain a new bound $c_i = n$;
3. Suppose that for groups R_i and R_{i+1} the bounds $d_i = n - 1, c_{i+1} = n + 1$, then R_i is replaced by $R_i \bigcup R_{i+1}$;
4. Otherwise, we create a new release group with the bounds $[n, n]$.

So we say that the released customer *forms* a release group.

We remark that in fact sets B and R change at discrete time moments on the scale counting all transitions of customers in the network including arrivals, departures, transitions between nodes. More precisely, the set B may change at moments of arrivals or transitions of customers between nodes (as one customer can collide with another at these moments). But the set R may change only at departure moments.

Definition. *Let $\{\tilde{t}_n\}_{n \geq 0}$ be departure times. If the departed customer forms the release group R_1 at a time $\tilde{t}_n$ and*

$$R_1 \bigcap B_1 = B_1, \tag{6}$$

then the moment of the next customer arrival to the network after $\tilde{t}_n$ is called the ***quasi-weak regeneration point****.*

That is, at the departure moment group R_1 includes group B_1. Group R_1 contains released customers beginning with number 1. This fact guarantees that after $\tilde{t}_n$ the state of the network process does not depend on customers with numbers $1, 2, \ldots, b_1$.

If there exist such groups $B_k (k > 1)$ for which $R_1 \bigcap B_k = B_k$ then they should be deleted (along with B_1) from the set B after the q-weak regeneration point is registered. Namely, such dependence groups that consist of numbers of the already released customers are retired. This situation arises when one or a few customers stay in the network for a long time and customers with higher numbers have already been serviced and have left the network.

We remark that for exact construction of weak regeneration points it is necessary to use the so-called barriers [4], which provide one-dependent regeneration cycle. Constructed q-weak regeneration points do not require this restriction. In this connection, q-weak regeneration generally leads to nonidentical distribution of the state of the network process at such q-weak regeneration points. At the same time barriers significantly reduce the number of regeneration points and impose restriction on a network configuration.

Our preliminary numerical results show that quasi-weak regeneration may be effective in simulation of large networks and has considerable advantage in the rate of confidence estimation in comparison with classic regeneration.

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