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Law of Large Numbers and Central Limit Theorem for Generalized Risk Processes[1]

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Let $N(t) = N_1(\Lambda(t)), t \geq 0$, be a Cox process controlled by some process $\Lambda(t)$ with nondecreasing right-continuous and almost surely finite sample paths starting from the origin. Here $N_1(t)$ is a Poisson process with unit intensity. Let $X_1, X_2, \dots$ be independent identically distributed positive random variables with $\mathsf{E}X_1 = a < \infty$. Assume that all the random variables and processes under consideration are independent.

A *generalized risk process* is a process of the form

$$R(t) = c\Lambda(t) - \sum_{j=1}^{N(t)} X_j, \quad t \geq 0, \tag{1}$$

with $c > 0$. If c is interpreted as premium rate, $N(t)$ is the number of insurance claims within the interval $[0, t]$ and $X_1, X_2, \dots$ are insurance claims, then the process (1) describes the surplus of an insurance company under stochastic fluctuations of risk (i. e., of the intensity of the flow of claims) and of portfolio (i. e., of the intensity of the flow of premiums).

The following result is a strengthening of a well-known result due to Lundberg (Lundberg, 1964) and can be regarded as the law of large numbers for process (1).

Theorem 1. *Assume that $\Lambda(t) \xrightarrow{P} \infty$ as $t \to \infty$ and $a > 0$. Let $D(t) > 0$ be an infinitely increasing function. Then*

$$\frac{R(t)}{D(t)} \Longrightarrow \text{(some) } Z \quad (t \to \infty)$$

if and only if there exists a nonnegative random variable U such that

$$\frac{\Lambda(t)}{D(t)} \Longrightarrow U \quad (t \to \infty).$$

In the latter case $Z \stackrel{d}{=} (c - a)U$.

We also present the central limit theorem-type results for generalized risk processes.

Theorem 2. *Assume that $\mathsf{E}X_1 = a \neq 0$, $0 < \mathsf{D}X_1 = \sigma^2 < \infty$ and $\Lambda(t) \xrightarrow{P} \infty$ as $t \to \infty$. Let $D(t) > 0$ be a function such that $D(t) \to \infty$ as $t \to \infty$. The one-dimensional distributions of the appropriately centered and normalized generalized risk process $R(t)$ converge weakly to the distribution of some random variable Z, that is,*

$$\frac{-R(t) - C(t)}{D(t)} \Longrightarrow Z \quad (t \to \infty)$$

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with real function $C(t)$ if and only if

$$\limsup_{t\to\infty}\frac{|C(t)|}{D^2(t)}\equiv k^2<\infty$$

and there exists a random variable V such that

$$Z\stackrel{d}{=}k\cdot\sqrt{\frac{a^2+\sigma^2}{|a-c|}}\cdot W+V$$

where W is the random variable with the standard normal distribution independent of V and

$$L_1\Big(\frac{(a-c)\Lambda(t)-C(t)}{D(t)},V(t)\Big)\longrightarrow 0\quad(t\to\infty)$$

where the distribution of the random variable $V(t)$ is defined by its characteristic function

$$\mathsf{E}\exp\{isV(t)\}=\exp\Big\{-\frac{s^2(a^2+\sigma^2)}{2|a-c|}\Big[k^2-\frac{|C(t)|}{D^2(t)}\Big]\Big\}\mathsf{E}\exp\{isV\},\quad s\in\mathbf{R}.$$

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