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A Family of Regenerative Moment Identities

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Let $X = (X_n : n \geq 0)$ be a positive recurrent and irreducible Markov chain taking values in a discrete state space S. Given a real-valued function $f : S \to \Re$, we consider the additive functional

$$S_n = \sum_{j=0}^{n-1} f(X_j)$$

induced by f. Suppose there exist (deterministic) constants α and σ^2 such that the central limit theorem

$$n^{-1/2}(S_n - n\alpha) \Longrightarrow \sigma N(0,1)$$

holds, as $n \to \infty$.

The constants α and σ^2 have regenerative representations. In particular, for $x \in S$, let $\tau(x) = \inf\{n \geq 1 : X_n = x\}$ be the duration of an "x-cycle". The parameters α and σ^2 may be expressed in terms of "regenerative cycle" - type quantities, namely

$$\alpha = \mathrm{E}_x \sum_{j=0}^{\tau(x)-1} f(X_j) / \mathrm{E}_x\, \tau(x) \tag{1}$$

$$\sigma^2 = \mathrm{E}_x(\sum_{j=0}^{\tau(x)-1} (f(X_j) - \alpha))^2 / \mathrm{E}_x\, \tau(x) \tag{2}$$

where $\mathrm{E}_x(\cdot) \triangleq \mathrm{E}(\cdot | X_0 = x)$. It has long been observed that (1) and (2) imply that the right-hand sides of (1) and (2) must be regenerative "invariants", in the sense that these expressions, involving moments over an x-cycle, must be independent of the choice of x.

In this paper, we show that the regenerative identities associated with (1) and (2) are just the first two such identities in an infinite sequence of such identities. The k'th such identity involves mixed moments of order at most k.

We also discuss generalizations of these ideas to Hariss recurrent Markov chains taking finite values in a general state space.