

Large Quantification by Stochastic Models

Elart von Collani

University of Würzburg,
Sanderring 2, D-97070 Würzburg, Germany
email: collani@mathematik.uni-wuerzburg.de

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In one of Vladimir Kalashnikov's last papers entitled "Quantification in Stochastics and the Stability Concept", he concludes:

The results reviewed above clearly show the tendency to getting quantitative results in stochastics by developing appropriate rigorous mathematical methods. This tendency can be traced back to Bernoulli, Laplace, Chebyshev, Lyapunov, Markov, Levy, Bernstein and other founders of the probability theory who stressed the necessity to investigate and assure quantitatively quality of models and methods. Analytical methods brought into the theory of probability stimulated enormous progress in obtaining deep qualitative purely mathematical results. They promoted abstract investigations but requirements of applications were often ignored. Simultaneously, interest to quantification of stochastic models was weakened. However, since 70s of the last century in the 2nd Millennium, this interest has grown up considerably. Due to this, various quantitative methods have been developed. It would be over-optimistic to say that all these methods have found applications. But they showed a maturity of the theory of probability, built a basis for future research, and, what is also important, showed the beauty of quantitative results. Now, in computer era, we have the opportunity not to restrict ourselves to only simple computational formulas. Almost any formula or algorithm are acceptable if we know that they yield practically useful and mathematically solid results. Therefore, it is absolutely actual to concentrate our efforts on developing quantitative methods resulting in appropriate numerical routines that force theoretical results working in applied areas.

Taking up Vladimir Kalashnikov's ideas, in this paper application-oriented requirements for the development of quantitative models are formulated and a novel approach in stochastics is suggested. It is outlined that this approach gives rise to new mathematical and computational problems which can be solved only by rigorously exploiting the computational power of modern computers. The result would be more appropriate and stable models making the laws based on limit theorems or purely mathematical derivation if not obsolete but less important.

The first part of the paper defines the terms *mathematics*, *science* and *stochastics*. Mathematics constitutes strictly formal logical systems, which are studies as such without concern of their meaning or usefulness. Science on the other hand means systematized investigation and knowledge of nature and the physical world. From the view point of science mathematics is the language for quantification and, therefore, indispensable for any *exact science*. Stochastics was inaugurated by Jakob Bernoulli about 300 years ago. It is the *science of prediction* and investigates the omnipresent real aspect of uncertainty whose sources are ignorance and randomness.

Unfortunately, stochastics in the sense of Jakob Bernoulli was abandoned in favor of two more or less independent disciplines namely *probability theory* and *statistics*. Probability theory is regarded as a special

branch of mathematics whereas statistics is looked upon as a mere methodology. Consequently, neither probability theory nor statistics were able to replace stochastics.

In the second part of the paper five quotations of Vladimir Kalashnikov are commented. The statement “Requirements of application were often ignored” is discussed and it is shown that traditional models do not take into due consideration ignorance and randomness by defining appropriate variables and functions.

Kalashnikov’s demand “Assure quantitatively quality of models and methods” addressing the quality of approximations is extended to the quality of usefulness of models.

The expression of the “Beauty of Quantitative Models” gives rise to the definition of a *Bernoulli Space* which should replace the concept of a probability field in stochastics. A Bernoulli Space describes very neatly any existing ignorance and expresses randomness in accordance to confirmed properties of the aspect of interest.

The quotation “Practical Useful and Mathematical Solid Results” is taken to stress the primary importance of *usefulness*, while mathematical correctness is only one possible means for arriving at useful results.

Demanding “Force Theoretical Results Working in applied Areas” means in its last consequence to build up an independent *science of stochastics* with the consequence that statistics as a mere methodology not rigorously based on theoretical foundations should be abandoned in its present form and probability theory should lose its ambiguous position between being a branch of mathematics and being a part of science.

Dealing scientifically with the aspect of uncertainty for the benefit of human societies, for science and for individuals has an ever increasing importance and, therefore, it is high time to re-establish the science of prediction called *stochastics* beside its mathematical counterpart *probability theory*.

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