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Matrix Queueing Theory

A.N. Dudin and V.I. Klimenok

Department of Applied Mathematics and Computer Science,
Belarusian State University, Minsk, Belarus
email: dudin@bsu.by, klimenok@bsu.by
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Traditional queueing theory deals mainly with one-dimensional stochastic processes like queue or orbit length, virtual or actual waiting or sojourn time, busy period, etc. Discrete or continuous time one-dimensional Markov chains and renewal theory are the main tools for their investigation.

However, the increasing complexity of the modern telecommunication networks (heterogeneous bursty correlated flows of information with the different requirements to Quality of Service, possibility of dynamic resources distribution, etc.) gave raise to investigating the queues whose behavior can be modelled in terms of multi-dimensional Markov chains. The important particular case of such Markov chains assumes that one component of the chain is denumerable (often this component represents queue or orbit length) while the other components are finite (the state of some external random environment, the states of processes which control the rates of input, service, breakdowns, disasters, vacations, searches, etc.).

Generally speaking, such a multi-dimensional Markov chain can be easily reduced to the usual one-dimensional Markov chain (e.g., just by enumeration the states in lexicographic order). However, following this way we lose the special structure (if it exists) of the one-step transition probability matrix and consequently lose the chance to investigate the chain analytically.

The alternative way for investigating the multi-dimensional Markov chain consists of dividing the state space of the chain to the infinite set of so called levels (each level corresponds to a fixed state of the denumerable component) with the finite number of states belonging to the level. In such a way, the stationary probabilities of the states belonging to the same level constitute a finite state vector while the transition probability matrix consists of matrices defining transitions between the levels together with transition of finite components. So, the equilibrium equations for original Markov chain can be rewritten in the form of system of linear equations for the stationary probability vectors. If the structure of this system is appropriate for deriving some analytic results, these results have a form which is a rather transparent matrix form of results that are valid for the corresponding queueing system with one-dimensional state space.

This way is related with the name of Marcel Neuts. In his book [12], he introduced into consideration and comprehensively investigated a special kind of two-dimensional continuous-time Markov chains (so called generalized Birth-and-Death processes and GI/M/1 type Markov chains). It allowed to get the stationary state distribution for a wide class of Markovian queues in the matrix geometric form. In his book [13], M. Neuts investigated the so called M/G/1 type Markov chains. It allowed him to get the transparent matrix analogs for known results for M/G/1 type queues (Pollaczek- Khinchine formula in particular) in case of versatile input or complex (e.g., semi-Markovian) service process. These works of M.Neuts (and his followers W. Ramaswami, S. Chakravarthy, D. Lucantoni, G. Latouche, etc.) and the works of G.P. Basharin (and his followers P.P. Bocharov, V.A. Naoumov, A.I. Gromov, etc.) created the background for developing a whole direction in applied probability called as Matrix Analytic Methods. This field of research has already own series of International conferences (the fourth one took place in July, 2002 in Adelaide, Australia).

The subfield of this direction belonging to the field of Queueing does not have own name yet while it evidently has his own specifics (comparing to ordinary Queueing), tools for investigation (combination of usual tools of Queueing Theory and methods of Matrix Theory), field of applications (modern telecommunication networks), set of involved researchers. The name offered to this direction in the present abstract is: the Matrix Queueing Theory. In contrast to classic Queueing Theory, the Matrix Queueing Theory deals with vectors of stationary probabilities and blocking transition probability matrices. The main tools for research are the multi-dimensional Markov chains and the theory of Markov renewal processes [6]. The obtained results formally can be considered as analytic ones and often (but unfortunately not always) have a form coinciding with the result for the corresponding standard queueing system. However, they usually contain some constant vectors which can be calculated only numerically (in the standard queues we have constants which are easily eliminated, e.g. from normalization condition). So, the essential part of research in the Matrix Queueing Theory is usually devoted to creating the effective algorithms for computing these constant vectors. The essential difficulty arising on this way stems from the fact that the matrices, which we would like to invert to get a nice and simple algorithm, are singular as a rule.

The literature in Matrix Queueing Theory is already extensive although the number of published books is not so big. In addition to M. Neuts' books [12,13], mention the book of Latouche, Taylor [8], proceedings [1,4,9,10] of four conferences in Matrix Analytic Methods and books in Russian by Bocharov and Pechinkin [3] and Dudin and Klimenok [7]. The list of active researchers in Matrix Queueing Theory almost coincides with the list of participants of conferences in Matrix Analytic Methods (see [1,4,9,10] for exhaustive list). B.D. Choi and his followers should be added to this list. Paper [5] by S.Chakravarthy can be treated as the first survey of results in the important subfield of the Matrix Queueing Theory - systems with the Batch Markovian Arrival Process (BMAP) [11]. Mention also the overviewing papers of W. Ramaswami in [4] and of M. Neuts [14] and the paper [2] by Baum and Kalashnikov where the notion of BMAP is generalized to the flows with arrivals distributed in space.

Matrix Queueing Theory seems to be one of the most promising, interesting and dynamically developing fields of Queueing Theory.

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