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Efficient Rare Events Simulation of Gaussian Processes

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1. INTRODUCTION

A key feature of modern broadband networks is their ability to provide stringent Quality of Service (QoS) guarantees to different classes of users. A primary QoS parameter is the Cell Loss Probability (CLP), whose typical values can be very small and therefore difficult to estimate through standard Monte Carlo (MC) simulation, since long run times are required to achieve accurate results. In this paper we propose two different techniques to efficently simulate a single server queueing system, loaded with Long Range Dependent (LRD) traffic. The two techniques rely on dynamic Importance Sampling (IS) and Hybrid Monte-Carlo (HMC) respectively. In this extended abstract we briefly describe the proposed approaches together with the main theoretical results achieved in this work (Section 2). In Section 3, we conclude by showing some preliminary simulation results.

2. SIMULATION APPROACHES

The basic issue addressed in this paper is the efficient simulation of a single server queue equipped with an infinite buffer. In particular, we focus on the estimation of the overflow probability, defined as the probability that the steady-state queue-length Q exceeds a given threshold b, which gives an upper bound for the CLP.

The input traffic is based on Fractional Brownian Motion (FBM), which has become a canonical model in the context of LRD traffic [5]. Let μ be the deterministic service rate, m the mean input rate and $X(t) = mt + B_H(t)$ the input process, where $B_H(t)$ is a zero-mean FBM of variance σ^2 and Hurst parameter H. Then the overflow event $\{Q > b\}$ can also be written

$$\{Q > b\} = A_\gamma := \{B_H(t) > \gamma_t : t \in [0, +\infty)\}$$

where $\gamma_t = b + t(\mu - m)$.

2.1. IS Approach

The basic idea underlying the IS technique is the *biaising* of the examined system, so that the target rare event becomes more likely to occur [2].

The efficiency of an IS-based algorithm depends on the choice of a proper change of measure to reduce the variance of the estimate. It is well known that the optimal change of measure (*zero-variance* pdf) involves the knowledge of the probability we want to estimate and therefore cannot be practically adopted. The issue is commonly tackled by minimizing some sub-optimal criteria instead. To this aim, we consider the class of pdf's that differ from the original one for the mean value only.

We first solve the minimum variance problem within this class of pdf's. We prove that, in the limit of Large Deviations, the optimal change of measure in this sense involves the concept of Most Likely Path (MLP) to overflow of FBM, already introduced in [4, 6].

Incidentally, we heuristically prove that in the case of large buffers the same result is obtained by solving for the pdf which minimizes the Cross Entropy [3] function with respect to the zero-variance pdf.

2.2. HMC Approach

An alternative approach can be derived by expressing the overflow probability as the expectation of a function of the MLP and of the Fractional Brownian Bridge (FBB) associated with a given instant of overflow. It can be shown that

$$\begin{aligned} P(Q > b) &= P[\sup_t (B_H(t) - \gamma_t) > 0] \\ &= E\left[\int_\Gamma^\infty e^{-\frac{x^2}{2\sigma^2 s^{2H}}} \frac{dx}{\sqrt{2\pi\sigma^2 s^{2H}}}\right], \end{aligned} \tag{1}$$

where Γ is the random quantity given by

$$\Gamma := \inf_t \frac{\gamma_t - Z_t}{\varphi(t/s)};$$

in the denominator we have the MLP $\varphi(t) = (1 + |t|^{2H} - |1 - t|^{2H})/2$, in the numerator the FBB $Z_t = B_H(t) - \varphi(t/s)B_H(s)$ which corresponds to a FBM conditioned to be in 0 at time s (note that we assume $E\left\{B_H(1)^2\right\} = \sigma^2$). The mean value in eq. (1) is then estimated by a MC approach, while the integral is performed exactly (or in general by numerical algorithms), hence the name Hybrid MC. The conditioning time s is a free parameter in the procedure, however we checked that every sensible choice of s gives comparable results as far as the variance of the estimator is concerned.

3. SIMULATION RESULTS

In this section we present some of the preliminary results achieved by applying the above described techniques. For IS we used the optimal change of measure as given by the MLP, while in the HMC tecnique we fixed the conditioning time to be the expected most likely time for overflow.

Figure 1 compares the estimates of the overflow probability obtained by means of the IS and HMC techniques with the following parameters $H = 0.8$, $m = 100$, $\mu = m/0.8$, $\sigma = 13$ with 1500 samples of length 400. For the sake of completeness, we also plot the analytic lower bound due to Norros [5].

For a fair comparison, in Figure 2 we show the standard deviations of the two estimators normalized by the overflow probability as given by the weighted mean of the estimates of the two different methods.

4. CONCLUSIONS

We propose two techniques for fast simulations of broadband communication systems. Our results are applicable to estimate the probability of rare events when modelling the broadband traffic using FBM. The efficient simulation algorithms proposed in this paper are based on IS and on a novel Hybrid MC technique.

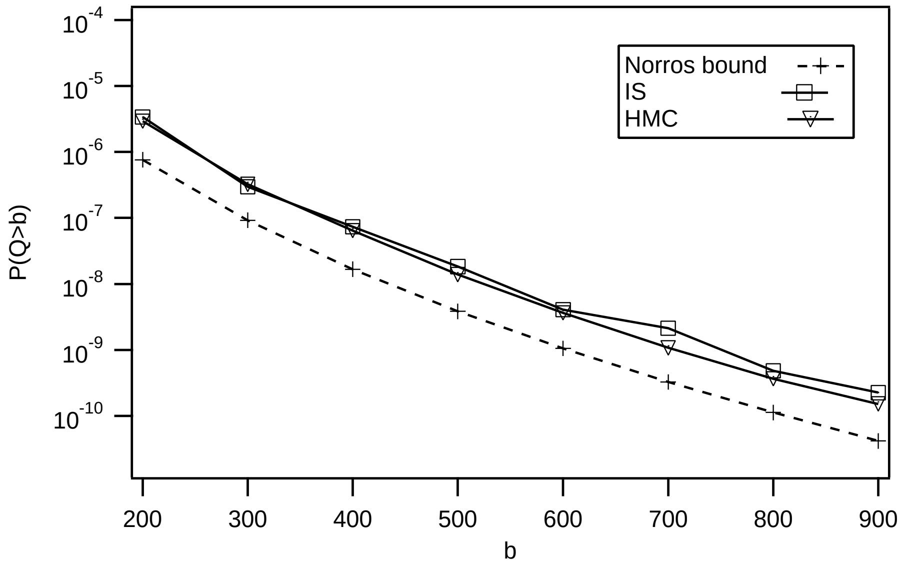

Figure 1. Overflow Probability estimates

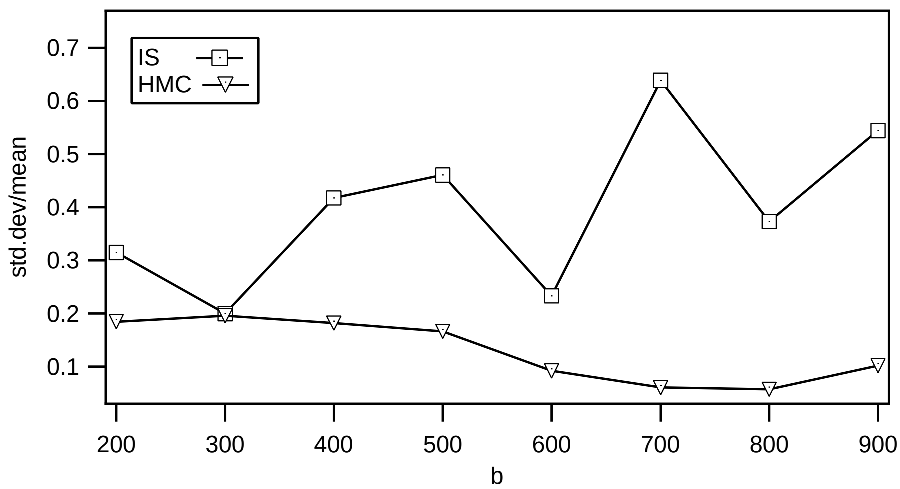

Figure 2. Accuracy of IS and HMC estimators

As shown by [1], IS has the main drawback of being sensible to the choice of tilted measure: a non optimal change of measure will often result in an exponentially growing variance of the estimator. As far as the HMC method is concerned, we can show that this drawback is not present, i.e. even a non optimal choice of HMC parameters (the conditioning time s) does not lead to a degradation of the estimator.

It is important to point out that the HMC method can be applied to any Gaussian process and in a wide variety of network systems (e.g. tandem queues, schedulers, etc.) and can be generalized with more than one conditioning or with dynamic choice of the parameters. Moreover it does not need any refined preventive theoretical analysis.

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