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Risk Theory and Geometric Sums

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Vladimir "Volodya" Kalashnikov was a most kind and considerate person. Although we certainly most remember his great personality, we all also knew him as a splendid researcher. I will consider, as mentioned in the title, risk theory and geometric sums which were interests we had in common. A main reference for geometric sums is Kalashnikov (1997).

Consider the classical model of an insurance risk business, i.e. where the claim occur according to a Poisson process $N = \{N(t);\, t \geq 0\}$ with intensity α and the costs of the claims are described by a sequence $\{Z_k\}_1^\infty$ of independent and identically distributed random variables, having a common distribution function F with mean μ.

The total amount of claims paid by the company in the interval $(0, t]$ is then described by the *claim process*

$$Y(t) = \sum_{k=1}^{N(t)} Z_k, \quad \left(\sum_{k=1}^{0} Z_k \stackrel{\text{def}}{=} 0 \right).$$

The *risk process*, X, is defined by

$$X(t) = ct - Y(t),$$

where c is a positive real constant corresponding to the premium income. We will here only treat the case with only positive risksums, i.e. we assume that $F(0) = 0$.

The *ruin probability* $\Psi(u)$ of a company facing the risk process X and having initial capital u is defined by

$$\Psi(u) = P\{u + X(t) < 0 \text{ for some } t > 0\}.$$

We have (this is, in fact, the *Pollaczek–Khinchine formula*)

$$\Psi(u) = \left(1 - \frac{\alpha\mu}{c}\right) \sum_{n=0}^{\infty} \left(\frac{\alpha\mu}{c}\right)^n \bar{F}_I^{n*}(u) = \frac{\rho}{1+\rho} \sum_{n=0}^{\infty} \left(\frac{1}{1+\rho}\right)^n \bar{F}_I^{n*}(u),$$

where

$$F_I(z) \stackrel{\text{def}}{=} \frac{1}{\mu} \int_0^z (1 - F(x))\, dx \quad \text{and} \quad \rho \stackrel{\text{def}}{=} \frac{c - \alpha\mu}{\alpha\mu}.$$

Thus we have an expression for the ruin probability which is a geometric sum.

Rather many years ago I got interested in "simple"approximations, by which we mean that the approximations only depend on some moments of F. The background of that interest was that some approximations, based on more or less ad hoc arguments had been proposed, which filled our requirement of being simple.

The best such approximation is, without any doubt the *De Vylder approximation*, proposed by De Vylder (1978). It is based on the idea to replace the risk process X with a risk process $\widetilde{X}$ with exponentially distributed claims such that

$$E[X^k(t)] = E[\widetilde{X}^k(t)] \quad \text{for } k = 1, 2, 3.$$

This leads to the approximation

$$\Psi_{\rm DV}(u) = \frac{3\zeta_2^2}{3\zeta_2^2 + 2\zeta_1\zeta_3\rho} \exp\left\{-\frac{6\zeta_1\zeta_2\rho u}{3\zeta_2^2 + 2\zeta_1\zeta_3\rho}\right\}$$

where

$$\zeta_k = E[Z_j^k], \quad k = 1, 2, 3.$$

Note that $\zeta_1 = \mu$.

Inspired by discussions with Volodya and his investigations in Kalashnikov (1997), I studied simple approximations (see [3]) and discussed how they might be analyzed.

A quite different reason for my interest in Volodya's work was that a geometric distribution may by regarded as a mixed Poisson distribution with exponential prior distribution. More precisely we consider a Poisson distribution where its mean λ is regarded as an exponentially distributed random variable. Such distributions, and the corresponding processes was considered in [2].

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