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Instability of Epsilon Deterministic Queueing Networks via the Fluid Model[1]

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1. INTRODUCTION

In this paper, we consider a multiclass queueing network model and the associated queue-length process. Finding conditions for stability (positive Harris recurrence or rate stability) of this queue-length process is known to be a non-trivial task since examples in the early 1990's demonstrated that the usual traffic conditions were not sufficient to insure stability for such systems (see [1], [7], [8], [11], [12]). A major breakthrough was made in a paper by Dai [4] (with refinements by Chen [3] and Stolyar [13], among others) in which the stability of the stochastic system was connected with the stability of a fluid model, which a continuous, deterministic analog to the original queueing network. Roughly speaking, Dai's result implies that a multiclass queueing network is stable if a corresponding mean-value fluid network analog is stable. The queueing network under consideration may be operating under a particular queueing discipline or a class of disciplines. Recent examples by Bramson [2] and Dai, Hasenbein, and VandeVate [6] have shown that the mean-value fluid model of a given stochastic network may not be sharp in determining stability under a given dispatch policy. This in turn implies that a general converse to Dai's theorem is not possible. However, specialized partial converses have been proven by Dai [5], Meyn [9], and Puhalskii and Rybko [10]. We consider a particular refinement of Dai's result which appeared in Chen [3]. In particular, Chen proved that under appropriate strong-law-of-large-number assumptions on the primitive processes, a multiclass queueing network is globally rate stable if the associated fluid model is globally weakly stable. In this paper, we prove a partial converse to Chen's theorem. In particular, we show that if the non-idling fluid model is not weakly stable, then there exists an $\epsilon > 0$ such that any ϵ-deterministic network is not globally rate stable. An ϵ-deterministic network can be thought of as a "nearly" deterministic system. We give an example in which the value ϵ can be calculated and a non-idling unstable policy can be identified. We also discuss potential extensions to this result to a larger class of networks. Below we provide more technical details of the main results.

2. DEFINITIONS AND RESULTS

We will not provide a detailed discussion of the multiclass queueing network, referring the reader to Dai [4]. Instead, we begin by defining a fluid model. The non-idling HOL *fluid model* of a multiclass

[1] This is joint work with David Gamarnik of IBM T.J. Watson Research Center, Yorktown Heights, New York, USA.

queueing network is defined through the following set of equations:

$$\bar{Q}(t) = \bar{Q}(0) + \alpha t + (P' - I)\bar{D}(t), \tag{1}$$

$$\bar{Q}(t) \geq 0, \tag{2}$$

$$\bar{D}(t) = M\bar{T}(t), \tag{3}$$

$$\bar{T}(0) = 0,\ \bar{T}(\cdot) \text{ is non-decreasing}, \tag{4}$$

$$\bar{Y}_j(t) = t - \sum_{k \in C(j)} \bar{T}_k(t),\ \text{non-decreasing} \tag{5}$$

$$\bar{Y}_j(t) \text{ can increase only when } \bar{W}_j(t) = 0 \tag{6}$$

The parameters in the fluid model are simply the mean value data taken from an associated multiclass queueing network. The column vector α is the set of exogenous input rates to the network. P is the routing matrix and M is a diagonal matrix with mean processing times for each class appearing on the diagonal.

We need the next several definitions to state our main results.

Definition 2.1. A pair of functions $(\bar{Q}(\cdot), \bar{T}(\cdot))$ on $[0, \infty)$ which satisfy (1)–(6) is called a non-idling fluid solution.

Definition 2.2. A fluid model is called *(globally) weakly stable* if all fluid solutions with $\bar{Q}(0) = 0$ have the property that $\bar{Q}(t) = 0$ for all $t \geq 0$.

Definition 2.3. A fluid model solution $(\bar{Q}(\cdot), \bar{T}(\cdot))$ is called linearly divergent if there exists an increasing sequence of time points $t_1 < t_2 < t_3 \dots$ such that

$$\liminf_{n \to \infty} \frac{|\bar{Q}(t_n)|}{t_n} = k > 0.$$

Definition 2.4. The random variable X with finite mean is said to be ϵ-deterministic if for every sample path $\big|X - \mathbb{E}[X]\big| \leq \epsilon$. The network is said to be ϵ-deterministic if all interarrival and service times are ϵ-deterministic.

The next two results are the main results of the paper, which provide a different kind of partial converse to Dai's stability result. More precisely, taken together Theorems 2.1 and 2.2 provide an exact converse, in the case of ϵ-deterministic networks, for a theorem in Chen [3] which states if the fluid model is globally weakly stable, then any associated queueing network is globally rate stable.

Theorem 2.1. *Consider the non-idling fluid model of an open multiclass queueing network. If the fluid model is not globally weakly stable, then there exists a non-idling solution which diverges linearly.*

Theorem 2.2. *Consider a non-idling fluid model. Suppose there exists a linearly divergent fluid solution. Then there exists an $\epsilon > 0$ such that any ϵ-deterministic network associated with the fluid model is globally unstable in the sense that*

$$|Q(t, \omega)| \longrightarrow \infty$$

on every sample path ω, for some non-idling policy. Furthermore, the number of jobs in the network diverges linearly. In particular, any associated deterministic network is globally rate unstable.

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