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The New Variant of Multivariate Generalization of the Generalized Poisson Distribution[1]

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When we consider the construction of the collective risk model the important step is the choice of the model for the process of claims. In classical risk model the homogeneous Poisson process with parameter λ is used. In this model it is assumed that all claims are the same type and only one claim can occur at a time. However, if we have claims of the different types, then we need to consider simultaneously several processes of claims. In this case several claims can occur at the same moment but they are of different types. Finally these processes can be dependent. So we face with the problem of the definition of multivariate Poisson distribution.

In our preceding paper ([1]) we have considered the class $\mathcal{F}_n$ of multivariate natural exponential families of probability distributions of random vector $X = (X_1, \ldots, X_n)$, every subvector of which has the distribution from the analogous family. Let the marginal distributions of this vector be the Poisson ones with parameters λ_k, $k = \overline{1, n}$. Then the joint distribution of random vector X is defined uniquely and has the following representation. For every collection $(i_1, \ldots, i_n)$, $i_k = 0 \vee 1$, $k = \overline{1, n}$, let $N_{i_1,\ldots,i_n}$ be independent random variables with Poisson distributions whose parameters are $\lambda_{i_1,\ldots,i_n}$. In what follows if we write $(i_1, \ldots, 1, \ldots, i_n)$ it means that we consider the index where the corresponding component is equal to 1. In our paper ([1]) we have shown that random vector X has the following representation:

$$X = \left(\sum_{1,i_2,\ldots,i_n} N_{1,i_2,\ldots,i_n}, \ldots, \sum_{i_1,\ldots,1,\ldots,i_n} N_{i_1,\ldots,1,\ldots,i_n}, \ldots, \sum_{i_1,\ldots,i_{n-1},1} N_{i_1,\ldots,i_{n-1},1} \right) ,$$

where $\lambda_k = \sum_{i_1,\ldots,1,\ldots,i_n} \lambda_{i_1,\ldots,1,\ldots,i_n}$. This variant is the generalization of the trivariate reduction method (see [2]).

The random vector X has another representation which is more useful for actuarial applications. Let ε_m be independent random vectors whose values are $(i_1, \ldots, i_n)$, $i_k = 0 \vee 1$, $k = \overline{1, n}$, which can occur with probabilities $p_{i_1,\ldots,i_n}$, and N be independent of $\{\varepsilon_m\}$ random variable with Poisson distribution with parameter λ. The random vector X has the following representation

$$X = \sum_{m=0}^{N} \varepsilon_m ,$$

where $\lambda \cdot p_{i_1,\ldots,i_n} = \lambda_{i_1,\ldots,i_n}$.

For a Poisson distribution the mean and variance are equal. Very often actuarial data indicate that this property fails. So we need to consider some other distributions. One of them is the so called generalized Poisson distribution. This is the distribution of a random variable Y which takes values $0, 1, 2, \ldots$ with

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probabilities

$$P(Y = m) = \frac{\lambda \cdot (\lambda + m \cdot \theta)^{m-1}}{m!} \cdot \exp(-\lambda - m\theta) \ ,$$

where $\lambda > 0, \max(-1, -\lambda/q) \leq \theta < 1$ and $q \geq 4$ is the largest positive integer for which $\lambda + \theta m > 0$ when $\theta < 0$ (see [3]). In what follows we consider for simplicity the case $0 \leq \theta < 1$. Multivariate generalizations of this distribution are considered and their properties are investigated in [4]–[7].

In our contribution we consider the new multivariate generalization of generalized Poisson distribution. Let $N_{i_1,\ldots,i_n}$ be independent random variables with generalized Poisson distributions with parameters $\lambda_{i_1,\ldots,i_n} \geq 0$ and $0 \leq \theta < 1$. By the definition the random vector $X = (X_1, \ldots, X_n)$ has a multivariate generalized Poisson distribution if it has the following representation:

$$X_k = \sum_{i_1,\ldots,1,\ldots,i_n} N_{i_1,\ldots,1,\ldots,i_n} \ .$$

Note that the above two construction for multivariate Poisson distribution lead now to different distributions.

In our contribution we consider the properties of multivariate generalization of the generalized Poisson distribution, the recursive formulas for probabilities evaluation, define the corresponding process of claims occurring and investigate its properties.

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