Информационные процессы, Том 2, № 2, 2002, стр. 204–205.

KALASHNIKOV MEMORIAL SEMINAR

Optimization Problems for the Parameters of Risk Processes[1]

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Received October 14, 2002

Assume that at the initial point of a time interval $[0, T]$ an insurance company has an initial capital u. Assume that $N(t)$, the number of insurance claims arrived up to time t, is the Poisson process with some intensity $\lambda > 0$. Let the insurance claims $X_1, X_2, \ldots$ be independent identically distributed random variables independent of the process $N(t)$. We consider the simplest case where the increase of the reserve due to insurance premiums is linear in time so that the total losses of the insurance company during the interval $[0, t]$ have the form

$$S(t) = \sum_{i=1}^{N(t)} X_i - \alpha\lambda t,$$

where $\alpha > 0$ is the premium rate. Then $R(t) = u - S(t)$ is the reserve of the insurance company at time t.

Let $c_1(t, u)$ be the costs at time t per money unit of the initial capital. We will assume that if $u \geq 0$, then $c_1(t, u) = c_1(t) > 0$. In this case the costs $c_1(t)$ are due to vain keeping money not invested. As $c_1(t)$, one may take, say, the profitability of derivative securities in which the insurance company could invest the money to get profit which is actually lost. And if $u < 0$ (this case corresponds to the situation where the company starts its insurance business having debts), then set $c_1(t, u) = -c_0(t) > 0$. Here $|c_0(t)|$ is the "penalty"for having debts. As $|c_0(t)|$, one may take, say, the bank rate under which the debts must be paid off. Let $c_2(t) > 0$ be the costs at time t per money unit due to the lack of money when a claim must be paid. As $c_2(t)$, one can take the riskless bank rate under which the company can take a credit to pay the claim. Then the total expected costs $D(u)$ of the insurance company during the period $[0, T]$ have the form

$$D(u) = u \int_0^T c_1(t, u)dt + \int_0^T c_2(t)\mathsf{E}\big(S(t) - u\big)^+ dt =$$

$$= \begin{cases} -u \int_0^T c_0(t)dt + \int_0^T c_2(t)\mathsf{E}\big(S(t) - u\big)^+ dt, & \text{if} \quad u \leq 0; \\ u \int_0^T c_1(t)dt + \int_0^T c_2(t)\mathsf{E}\big(S(t) - u\big)^+ dt, & \text{if} \quad u \geq 0, \end{cases} \tag{1}$$

where, as usual, $x^+ = \max\{x, 0\}$.

We show that the optimal value u_0 which minimizes $D(u)$ satisfies the equation

$$\int_0^T c_2(t)\mathsf{P}(S(t) \geq u)dt = \int_0^T c_1(t)dt. \tag{2}$$

[1] Research supported by the Russian Foundation for Basic Research, project 00–01–00360, and by the Russian Humanitarian Scientific Foundation, project 00–02–00152a.

We give asymptotic estimates for u_0 depending only on three first moments of X_1 and present guaranteed estimates for u_0 in the case of uniformly bounded claims.

For example, assume that $c_1(t) \equiv c_1$, $c_2(t) \equiv c_2$. Denote

$$\delta = 1 - \frac{c_1}{c_2}, \qquad m = \mathsf{E}X_1.$$

We show that if $c_1 > c_2$, then $u_0 = 0$ and otherwise the following result takes place.

THEOREM 1. *Assume that $X_i \leq H$ for some finite $H > 0$ and*

$$\big(1 - e^{w\lambda T}\big) H \log \delta \geq w(m - \alpha)(\lambda T)^2.$$

Then

$$u_0 \geq u_1 = H \log \delta \cdot \frac{1 - e^{w\lambda T}}{w\lambda T}$$

where $w = (e-2)H^{-2}\mathsf{E}X_1^2 - (\alpha - m)H^{-1}$.

We also give guaranteed estimates for the parameters α and T providing the conditions $\mathsf{P}\big(R(T) \geq R_0\big) \geq \gamma$ and $\mathsf{E}R(T) \geq R_0$ under the optimal choice of u with given $R_0 > 0$ and $\gamma \in (0,1)$.