Информационные процессы, Том 2, № 2, 2002, стр. 214–215.

KALASHNIKOV MEMORIAL SEMINAR

On Sub-Exponential Mixing Rate for a Class of SDEs[1]

S.A. Klokov* and A.Yu. Veretennikov**
**School of Mathematics University of Leeds*
Woodhouse Lane, LS2 9JT Leeds, UK (current address)
klokov@maths.leeds.ac.uk
Institute of Mathematics Omsk, Russia (on leave)
***Department of Mathematics Kansas State University*
Lawrence, KS, 66044 USA (current address)
veretennikov@math.ukans.edu
School of Mathematics University of Leeds, UK (on leave)
Institute of Information Transmission Problems,
19 B.Karetnii, 101447, Moscow, Russia
Received October 14, 2002

1. INTRODUCTION

We establish sub-exponential bounds for the β-mixing rate and for the rate of convergence to the invariant measure for a class of diffusion processes satisfying a stochastic differential equation in R^d,

$$X_t = x + \int_0^t b(X_s)ds + W_t, \quad t \geq 0.$$

Here b is a Borel d-dimensional bounded function, W_t – d-dimensional Wiener process. We consider a strong solution, that is, $X_t \in F_t^W$ for any $t \geq 0$.

Sub-exponential mixing bounds are used in the asymptotic analysis of diffusion processes, in particular, they are useful in moderate deviation theorems.

Sub-exponential convergence were studied in [1], sub-exponential mixing bounds in [4].

The method we use was suggested in [6] for studying polynomial mixing and convergence bounds, and then adjusted for sub-exponential case in [4]; we give better bounds under same conditions in compare to the latter paper; the essential novelty is that when $p \to 0$, one can choose α and β arbitrarily close to 1 (see below), i.e. the bounds are approaching exponential ones.

Mixing bounds for Markov processes are based on analysis of recurrence properties. This is the area where very essential results were established by V. V. Kalashnikov [2], [3].

2. MAIN RESULT

We assume that

$$(H_b) \qquad (b(x), x/|x|) \leq -r|x|^{-p}, \quad 0 < p < 1, \ r > 0.$$

The value $p = 0$ corresponds to exponential (see [5]), and $p = 1$ to polynomial beta-mixing (see [6]). Below μ_t^x is the law of X_t, μ is its invariant measure, var is the total variation metric, $\tau = \inf(t \geq 0 : |X_t| \leq M)$, and

$$\beta^x(t) = \sup_{s \geq 0} E_x var_{F^X_{\geq t+s}} (P(B|F^X_{\leq s}) - P(B)).$$

[1] To the memory of V. V. Kalashnikov.

Theorem. *Let* (H_b) *hold true,* $\beta < 1 - p$, $\alpha < \beta/(1+p)$. *Then there exist positive constants* $c_0 - c_8$, *and* M *large enough such that*

$$E_x e^{c_1 \tau^\alpha} \le c_0(1 + e^{c_2|x|^\beta}),$$

$$\sup_{t\ge 0} E_x e^{c_3|X_t|^\beta} \le c_0(1 + e^{c_4|x|^\beta}),$$

$$var(\mu_t^x - \mu) \le c_0(1 + e^{c_5|x|^\beta})\, e^{-c_6 t^\alpha},$$

$$\beta_t^x \le c_0(1 + e^{c_7|x|^\beta})\, e^{-c_8 t^\alpha}.$$

3. METHOD

We use "quasi-Lyapunov functions" $\exp(c(1+t)^\alpha)(1+t)^{-(1-\alpha)}\exp(c'|X_t|^\beta)$.

This term means that the Lyapunov property is "almost satisfied"; the remainder possesses a bound sufficient to show sub-exponential recurrence.

4. ACKNOWLEDGEMENTS

This work was written to revere V. V. Kalashnikov and his great contribution to stochastic analysis. It was supported by the grants EPSRC-GR/R40746/01 for both authors, and INTAS-99-0590 and RFBR-00-01-22000 for the second author.

REFERENCES

1. Ganidis, H.; Roynette, B., Simonot, F. Convergence rate of some semi-groups to their invariant probability. *Stochastic Process. Appl.*, 1999, 79, no. 2, pp. 243–263.
2. Kalashnikov, V. V. The property of γ-reflexivity for Markov sequences. *Sov. Math., Dokl.*, 1973, 14, pp. 1869–1873; transl. from *Dokl. Akad. Nauk SSSR*, 1973, 213, pp. 1243–1246.
3. Kalashnikov, V. V. *Qualitative analysis of the behavior of complex systems by the method of test functions.* Moscow: Nauka, 1978 (Russian).
4. Malyshkin, M. N. Subexponential estimates of the convergence rate to the invariant measure for stochastic differential equations (Russian), *Teor. Veroyatnost. i Primenen.*, 2000, 45, no. 3, pp. 489–504.
5. Veretennikov, A. Yu. Bounds for the mixing rate in the theory of stochastic equations. *Theory Probab. Appl.*, 1987, 32, no.2, pp. 273–281; transl. from *Teor. Veroyatn. Primen.*, 1987, 32, no.2, pp. 299–308.
6. Veretennikov, A. Yu. On polynomial mixing bounds for stochastic differential equations. *Stochastic Process. Appl.*, 1997, 70, no. 1, pp. 115–127.