Информационные процессы, Том 2, № 2, 2002, стр. 226–227.

Liquid Limit and Entropy

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Received October 14, 2002

Let us have a finite set $T = \{q\}$ of states and N particles, any of which can be in one of these states. Suppose, that during the time Δt any pair of particles, which are in the states q, q_1, interacts and transits to states q', q'_1 with the probability $\frac{1}{N} W(q', q'_1 | q, q_1)\Delta t$. The following theorem holds.

Theorem 1. *Let for $t = 0$ the limits exist*

$$C_q(0) = \lim_{N\to\infty} \frac{n_q(0; N)}{N},$$

where $n_q(0; N)$ - number of molecules in the state q for $t = 0$. Then for any q and any $t > 0$ there exists a limit in probability

$$C_q(t) = \lim_{N\to\infty} \frac{C_q(t; N)}{N}$$

and differential equations hold

$$\frac{dC_q(t)}{dt} = \sum_{q_1, q', q'_1} W(q, q_1 | q', q'_1) C_{q'}(t) C_{q1'}(t) -$$

$$- \sum_{q_1, q', q'_1} W(q', q'_1 | q, q_1) C_q(t) C_{q_1}(t). \qquad (1)$$

Suppose that for transition probabilities the following condition (Stuckelberg condition [1]) holds: There exists a positive measure $m_0(q)$ on the set T such that

$$\sum_{q', q'_1} W(q', q'_1 | q, q_1) m_0(q) m_0(q_1) = \sum_{q', q'_1} W(q, q_1 | q', q'_1) m_0(q') m_0(q'_1).$$

Then the following theorem holds.

Theorem 2. *The relative entropy*

$$H(C) = \sum_q C_q ln \frac{m_0(q)}{C_q}$$

is the Lyapunov function for differential equations (1), i.e.

$$\frac{dH(C(t))}{dt} >= 0,$$

and

$$\frac{dH(C(t))}{dt} = 0$$

only in the case when

$$\frac{dC(t)}{dt} = 0.$$

Denote by μ_N the stationary distribution of our Markov process. From Theorems 1 and 2 the next theorem follows.

Theorem 3. *Any limit point of the sequence of measures μ_N is concentrated on the set of $C = \{C_q\}$, which are the roots of the right-hand side of equations (1).*

REFERENCES

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