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Weak Regeneration for Modeling of Queueing Processes[1]

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We discuss new possibilities which the weakly regenerative approach (when a dependence between two adjacent regeneration cycles is allowed) opens in modeling and simulation of queueing network processes.

First we extend the class of regenerative inputs constructing weak regeneration for a superposition of n independent, stationary renewal processes in which case generally, classical regeneration does not exist. We assume that process i is generated by the i.i.d. interrenewal times $\{\xi_n^{(i)}\}_n$ with d.f. F_i and expectation $a_i \in (0,\infty), i = 1, \ldots, n$. Let $\xi_i(t),\ (\tilde{\xi}_i(t))$ be right-continuous unfinished (attained) renewal time at instant t in the renewal process i. By the stationarity,

$$\mathsf{P}(\xi_i(t) \le x) = \mathsf{P}(\tilde{\xi}_i(t) \le x) = \frac{1}{a_i}\int_0^x (1 - F_i(u))du,$$

for all $i,\ t \ge 0,\ x \ge 0$. Fix arbitrary $a > 0$ such that $\min_i(1 - F_i(a)) > 0$, and introduce an increasing sequence of the instants

$$T_0^{(a)} = 0,\ T_{n+1}^{(a)} = \inf_{t\ge 0}\Big(t:\ t \ge T_n^{(a)},\ \xi_i(t) \le a,\ i = 1,\ldots,n\Big) + a,\ n \ge 0.$$

Define Markov process $\xi(t) = \Big(\xi_1(t),\ldots,\xi_n(t)\Big)$ and let

$$G_n^{(a)} = \Big\{\xi(t) : T_n^{(a)} \le t < T_{n+1}^{(a)};\ \ T_{n+1}^{(a)} - T_n^{(a)}\Big\},\ n \ge 0.$$

We show that instants $\{T_n^{(a)}\}$ form an embedded renewal process of weak regeneration points of the process $\xi = \{\xi(t),\ t \ge 0\}$ with one-dependent regeneration cycles $\{G_n^{(a)}\}$ and the i.i.d. cycle lengths $\{T_{n+1}^{(a)} - T_n^{(a)},\ n \ge 0\}$. Let $\xi_i(T_m^{(a)}) = \xi_m^{(i)},\ \xi_m = (\xi_m^{(1)},\ldots,\xi_m^{(n)}),\ m \ge 0$.

It is shown that distribution of ξ_m (*regeneration measure*) has the form $H^{(a)} = \prod_i H_i^{(a)}$ and independent of the sequence $\{T_n^{(a)}\}$ and index m. At that marginal distributions

$$H_i^{(a)}(x) = \mathsf{P}(\xi_m^{(i)} \le x) = \frac{\int_0^a (F_i(x+y) - F_i(y))dy}{\int_0^a (1 - F_i(x))dx},\ \ i = 1,\ldots,n.$$

In particular, it then follows that we need not randomization based on the splitting technique to construct (weak) regeneration for the superposition of stationary renewal processes, see [2], [12].

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By analogy, we construct a weakly regenerative structure for the multi-dimensional workload process describing a multiserver queue $GI/G/m/0$ with losses and a queue $GI/G/\infty$ with infinite number of servers.

These new constructions of regenerative queueing processes lead to the necessity to analyze (regenerative) queues and networks with a weakly regenerative input. In this regard, we discuss an approach to stability of such processes based on a characterization of a renewal process by means of the limit behaviour of the unfinished renewal time and an "unloading"procedure, [4]. Then we touch upon the decomposition of this class of the regenerative networks (this problem was discussed in [3] for a narrower class of the networks).

Also we consider a simulation aspect. The great advantage of the regenerative approach is based on a possibility to apply (in a modified form) well-developed procedures of classical statistics to correlated data, [5], [6], [10],[11].

In the typical network setting a regeneration event occurs if the network process hits zero (under classical regeneration) or a neighborhood of zero (under weak regeneration), [6].

Although confidence intervals based on different types of regeneration points are asymptotically equivalent, the difference in required simulation time is often crucial for the efficiency of simulation procedure, [5], [7], [8], [10], [11]. The regenerative approach first was applied to simulation of the stochastic systems where the frequency of classical regeneration points was sufficient to estimate the process characteristics in acceptable simulation time, [10], [11]. But in modern communication networks such points occur as a rule too rarely, or not at all, which precludes their use in actual simulation. We discuss the conditions when weakly regenerative simulation of a queueing network is extremely effective (whereas classical regeneration is not). This new approach turns out to be especially useful for a large network with moderate/light traffic rates. Moreover, in a weak regeneration case monotonicity of some important network processes (workload, sojourn time) allows to increase estimation precision based on the specific variance reduction technique, [7].

We discuss also a possibility to apply the regenerative simulation to estimate the steady-state characteristics of some long-range dependent queueing processes. It is known that waiting time sequence $\{W_n\}$ in the stationary queue $GI/G/1$ with service time S and interarrival time τ is long-range dependent i.e.

$$\sum_{n=1}^{\infty} cov(W_0, W_n) = \infty,$$

if

$$\mathsf{E}\tau < \infty,\ \mathsf{E}S^3 < \infty,\ \mathsf{E}S^4 = \infty,$$

see [9]. At the same time, under these assumptions the busy period length α is such that $\mathsf{E}\alpha^3 < \infty$. This implies that unfinished renewal time $\tilde{\alpha}$ at instant t (time until next regeneration point under stationarity) has finite second moment, and hence,

$$\mathsf{P}(\tilde{\alpha} > x) = o(x^{-2}) \text{ as } x \to \infty. \tag{1}$$

We note that the same assumption $\mathsf{E}S^3 < \infty$ implies finiteness of the 3-rd moment of the idle period which is included in regeneration cycle length.

It follows that the regenerative method can be successfully applied for the actual simulation and estimation of the long-range dependent waiting time process. We conjecture that these observations hold true for a wider class of queueing processes and give a new promising opportunity for the application of the regenerative approach to analyze modern communication networks.

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