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Prosperity Paradox of Risk Theory

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Consider the risk process

$$x(t) = x + ct - \sum_{j=1}^{N(t)} \eta_j,$$

which appears within the framework of the so-called Cramer-Lunberg insurance risk model. It is assumed that

- $\eta_j,\ j = 1, 2, \dots$, are positive i.i.d. variables having common non-lattice distribution function;
- $N(t) = \sup(n \geq 1:\ T_n < t)$, where $\{T_n\}$ is the Poisson point process on the half-line $(0, \infty)$ that is

$$\tau_1 = T_1,\ \tau_j = T_j - T_{j-1},\ j = 2, 3, \dots,$$

  are i.i.d. variables having common exponential distribution with scale parameter $\sigma > 0$;
- the sequences $\{\eta_j\}$ and $\{\tau_j\}$ are independent of each other;
- $x \geq 0$ and $c > 0$ are constant.

The variables $\{\eta_j\}$ are interpreted as *claim sizes*. The claim of the size η_j occurs at the instant of time $t = T_j$. So, $\tau_j,\ j = 2, 3, \dots$, stand for *inter-arrival times*. Then $N(t)$ denotes the total number of claims arrived up to the instant of time t. Obviously, $N(t),\ 0 < t < \infty$, is a homogeneous Poisson point process with intensity $\lambda = 1/\sigma$. So,

$$P(N(t) = k) = \frac{(\lambda t)^k}{k!} e^{-\lambda t},\ k = 0, 1, 2, \dots .$$

Furthermore, x stands for the *initial capital* while c is called *gross premium rate* or *premium income rate*.

The *ruin probability* is defined as

$$\psi(x) = \mathrm{P}(\inf_{t>0} x(t) < 0).$$

Consider the random walk $\zeta_0 = 0,\ \zeta_1,\ \zeta_2, \dots$ generated by the successive sums $\zeta_n = \sum_{j=1}^{n} \xi_j$ where $\xi_j = \eta_j - c\tau_j$. Since the evolution of $x(t)$ within (T_{j-1}, T_j) is deterministic

$$\psi(x) = \mathrm{P}(\bar{\zeta} = \sup_{n \geq 0} \zeta_n > x).$$

Thus, evaluation of the ruin probability can be reduced to a boundary problem of a random walk.

Assume that $E\eta_1 = \mu < \infty$. Then $E\xi_1 = \mu - c\sigma = \mu - c/\lambda$. IF $E\xi_1 \geq 0$ then $\mathrm{P}(\bar{\zeta} > x) = 1$. So, the problem of ruin probability evaluation is not trivial provided $E\xi_1 < 0$ or $c > \frac{\mu}{\sigma} = \lambda\mu$. The last condition is called *safety loading condition*.

The theory related to the ruin probability evaluation is rather advanced (see e.g. Asmussen (1987), Grandell (1991), Embrecht et al. (1997), Kalashnikov (1997)). As to statistical aspects of the theory they are much less developped. Here the recent works of Bening and Korolev (1999, 2000a, 2000b) should be mentioned.

From the view-point of practice the following set-up seems reasonable. At the initial moment $t = t_0$ no information about the distributions of $\tau = \tau_j$ and $\eta = \eta_j$ and is available. Having observed $t_1, t_2, \ldots, t_n$ and $y_1, y_2, \ldots, y_n$ one can make inference on basic properties of the risk process provided no ruin took place until $t = T_n$. However, at this moment the total capital equals $x - \zeta_n$. So, it is natural to make inference on $\psi(x - \zeta_n)$ but not on $\psi(x)$. Our basic goal is to show that usually the data accumulated within the time interval $(0, T_n)$ do not allow one to make reasonable inference about $\psi(x - \zeta_n)$. So one encounters a paradoxical situation: *the richer the insurer the less he knows about his true risk.*

REFERENCES

Asmussen, S. (1987) *Applied Probability and Queues.* J. Wiley, Chichester etc.

Bening, V.E. and Korolev, V.Yu. (1999) Statistical estimation of ruin probability for generalized risk process. *Theory Probab. Appl.* **44**, 161-164.

Bening, V.E. and Korolev, V.Yu. (2000a) *Introduction to the Mathematical Risk Theory.* MAKS Press, Moscow(*in Russian*)

Bening, V.E. and Korolev, V.Yu. (2000b) *Generalized Risk Processes.* MAKS Press, Moscow(*in Russian*)

Embrechts, P., Kluppelberg, C. and Mikosch, T. (1997) *Modelling Extremal Events for Insurance and Finance.* Springer, Berlin.

Grandell, J. (1991) *Aspects of Risk Theory.* Springer, New York etc.

Kalashnikov, V. V. (1997) *Geometric Sums: Bounds for Rare Events with Applications.* Kluver Acad. Press, Dordrecht.