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On a Storage Model with Local Time Input

I. Norros[*] and P. Salminen[**]

[*] *VTT Information Technology*
FIN-02044 VTT, Finland
email: ilkka.norros@vtt.fi
[**] *Mathematical Department,Abo Akademi University,*
FIN-20500 Turku, Finland
email: phsalmin@abo.fi
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Let $\{X_t^{(1)} : t \geq 0\}$ and $\{X_t^{(2)} : t \geq 0\}$ be two independent reflecting Brownian motions on $[0, +\infty)$ with drift $-\mu < 0$. Define

$$Y(t) := \begin{cases} X_t^{(1)}, & \text{t>0} \\ X_{-t}^{(2)}, & \text{t<0} \end{cases}$$

and, further, let $Y(0)$ be an exponentially with parameter 2μ distributed random variable and set

$$X_0^{(1)} = X_0^{(2)} = Y(0). \tag{1}$$

We say that Y is a reflecting Brownian motion on $[0, +\infty)$ with drift $-\mu < 0$ in stationary state. Introduce, under the condition (1), the local time processes of $X^{(1)}$ and $X^{(2)}$ as follows

$$L_t^{(i)} := \lim_{\varepsilon \to 0} \frac{1}{2\varepsilon} \int_0^t \mathbf{1}_{[0,\varepsilon)}(X_s^{(i)})\, ds, \tag{2}$$

for $i = 1, 2$, respectively. Recall that the limit in (2) exists a.s. Finally, define

$$L_t := \begin{cases} L_t^{(1)}, & t \geq 0, \\ -L_{-t}^{(2)}, & t \leq 0. \end{cases}$$

Our main object is to study the properties of the process

$$S_t := \sup_{u \geq t}(L_u - L_t - (u - t)), \quad t \in \mathbf{R}.$$

Referring to Reich's formula we call $S = \{S_t : t \in \mathbf{R}\}$ a storage process with local time input, service rate 1, and unbounded buffer associated to reflecting Brownian motion with drift $-\mu$. Informally, S_t is the size of the storage at time $-t$.

In this talk we discuss the following results

1. S is a stationary process in stationary state if and only if $0 < \mu < 1$. The stationary distribution is given (for all $t \in \mathbf{R}$) by

$$\mathbf{P}(S_t > a) = \mu\, \mathrm{e}^{-2(1-\mu)a}, \quad a \geq 0,$$

   and, consequently, $\mathbf{P}(S_t = 0) = 1 - \mu$.
2. Let

$$g_i := -\inf\{t > 0 : S_t > 0\} \quad \text{and} \quad d_i := -\sup\{t < 0 : S_t > 0\}.$$

These are called, in case $S_0 = 0$, the starting time and the ending time, respectively, of the on-going idle period (observed at time 0). The joint distribution of g_i and d_i given that $S_0 = 0$ has the following Laplace transform

$$\mathbf{E}\big(e^{\alpha g_i - \beta d_i} \mid S_0 = 0\big) =$$

$$\frac{8\mu}{\big(\sqrt{2\alpha+\mu^2}+2-\mu\big)\big(\sqrt{2\beta+\mu^2}+2-\mu\big)\big(\sqrt{2\alpha+\mu^2}+\sqrt{2\beta+\mu^2}\big)},$$

and, in particular, $-g_i$ and d_i given that $S_0 = 0$ are identically distributed.

3. Let

$$g_b := -\inf\{t > 0 \,:\, S_t = 0\} \quad \text{and} \quad d_b := -\sup\{t < 0 \,:\, S_t = 0\}.$$

These variables are called, in case $S_0 > 0$, the starting time and the ending time, respectively, of the on-going busy period (observed at time 0). It is shown that $-g_b$ and d_b are identically distributed given $S_0 > 0$ with

$$\mathbf{E}(\mathrm{e}^{\alpha g_b} \mid S_0 > 0) = \mathbf{E}(\mathrm{e}^{-\alpha d_b} \mid S_0 > 0) = \frac{2(1-\mu)}{\sqrt{2\alpha + (1-\mu)^2} + \alpha + 1 - \mu}.$$

4. The paths of the input process of the storage, L, have the non-degenerate multifractal spectrum

$$f^*(h) = \begin{cases} 0, & h \in (0, \frac{1}{2}), \\ \frac{1}{2}, & h \in [\frac{1}{2}, \infty). \end{cases}$$

Note that the spectrum does not depend on μ. Thus, our system provides an example showing that multifractal character of an input process can be compatible with an exponential storage level distribution, and that the spectrum has little or nothing to do with queueing behavior.

The talk is based on [1], which paper can be seen as a continuation to an earlier paper [2].

REFERENCES

1. Mannersalo, P., Norros, I., Salminen, P. (2002) *On a storage process with local time input*, Preprint.

2. Salminen, P., Norros, I. (2001) *On busy periods of the unbounded Brownian storage.* Queueing Systems 39, 317-333.