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Efficient Computations of Body Moments

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It is well known that such important geometric characteristics of 3D bodies as volume and orientation can be defined in terms of moments. A moment computation depends greatly on a body representation. The most popular representation is a polygonal representation. In this case an object is represented by a mesh of polygonal facets.

The report presents explicit formulae for calculation of volume and surface moments for 3D polyhedral shapes. The formulae can be generalized for polytopes in $\mathbb{R}^n$. We discuss also efficient algorithms for calculation of 3D body volume and surface moments. The algorithms are based on the proposed formulae and take advantages of a polygonal representation. They use only coordinates of body vertices and faces orientation.

The way to compute a moment of a polyhedral shape is to compute first the moment of a tetrahedron with one vertex in the origin. We derive the formula for volume moments of arbitrary order using the Dirichlet integral and a linear substitution transforming an arbitrary tetrahedron to the coordinate tetrahedron. To compute a surface moment on a tetrahedron face we also use the Gauss-Ostrogradsky formula.

A moment of an arbitrary polyhedron is computed as the sum of oriented moments of tetrahedra with one vertex in the coordinate origin and the opposite face constructed by a triangulation of every face of a polyhedron. This procedure and the resulting formula does not depend on the mutual position of vertices and faces of a polyhedron.

Let P is a 3D polyhedron with n faces, and face i has s_i vertices $(x_{i,0}, y_{i,0}, z_{i,0}), \ldots, (x_{i,s_i-1}, y_{i,s_i-1}, z_{i,s_i-1})$ numbered in a counter-clockwise order with respect to the outer normal. Then the following formula is true for a volume moment $m_{pqr}V$ of order $p+q+r$ [1]:

$$m_{pqr}V(P) = \sum_{i=1}^{n} \sum_{j=1}^{s_i-2} \begin{vmatrix} x_{i,0} & x_{i,j} & x_{i,j+1} \\ y_{i,0} & y_{i,j} & y_{i,j+1} \\ z_{i,0} & z_{i,j} & z_{i,j+1} \end{vmatrix} \frac{\Sigma_{pqr}}{(p+q+r+3)!},$$

where

$$\Sigma_{pqr} = \sum_{k_1+k_2+k_3=p} \; \sum_{l_1+l_2+l_3=q} \; \sum_{m_1+m_2+m_3=r} \frac{p!q!r!(k_1+l_1+m_1)!(k_2+l_2+m_2)!(k_3+l_3+m_3)!}{k_1!k_2!k_3!l_1!l_2!l_3!m_1!m_2!m_3!} x_{i,0}^{k_1} x_{i,j}^{k_2} x_{i,j+1}^{k_3} y_{i,0}^{l_1} y_{i,j}^{l_2} y_{i,j+1}^{l_3} z_{i,0}^{m_1} z_{i,j}^{m_2} z_{i,j+1}^{m_3},$$

The surface moment $m_{pqr}S$ equals [2]

$$m_{pqr}S(P) = \sum_{i=1}^{n} \sum_{j=1}^{s_i-2} 2A_{i,j} \frac{\Sigma_{pqr}}{(p+q+r+2)!},$$

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where $A_{i,j}$ is the area of the triangle with vertices $(x_{i,s}, y_{i,s}, z_{i,s})$, $s = 0, j, j+1$.

The similar formulas are true for n-dimensional case.

Our implementation of the algorithms for polyhedra moments is written in C++. The set of faces is considered as a sequence, or a "stream." A body moment is computed as the sum of corresponding moments calculated in several computational threads, running in parallel, each of them processing independently a subsequence of faces. The algorithm computes volume and surface moments efficiently in a serial or parallel way.

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