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WORKSHOP ON IMAGE PROCESSING AND RELATED MATHEMATICAL TOPICS

Gibbs Classifiers and Fast Methods Their Determination

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New classifiers for models with Gibbs prior distributions of exponential and Gaussian types are presented. It is supposed observations are characterized by a feature function taking finite number of arbitrary rational values. The clusters of homogeneous observations are described by labels with values in another set of rational numbers. They are assumed to be dependent and distributed according to either the exponential or the Gaussian Gibbs law. Instead traditionally used local neighborhoods of nearest observations the completely connected graph of dependence of all observations is employed for the Gibbs prior distributions.

The methods of finding the classifiers by the reduction of the problem of identification of the optimal Bayes decision rule (OBDR) to the problem of determination of the minimum cut of an appropriate network are developed. The methods allow fast identification of the OBDR for large samples (in particular, they answer the question posed by Greig Porteous and Seheult in [1]).

For the models with exponential and Gaussian Gibbs prior distributions the case of rationally valued feature functions and cluster labels is reduced to the case of integer classification. So, let observations $\mathbf{y} = (y_1, y_2, \dots, y_n)$ be with the feature function f that takes finite number integer values in $Z_L = \{0, 1, \dots, L\}$ and let $f_i = f(y_i)$. Let $\mathcal{M} = \{m(1), \dots, m(k)\}$, $(0 \le m(1) < m(2) < \dots < m(k) \le L)$ be a set of allowable cluster labels. Suppose that observations $y_1, y_2, \dots, y_n$ belong to k clusters and each cluster is specified by integer number j, $(1 \le j \le k)$, as well as by an appropriate fixed integer label $m_j \in \mathcal{M}$.

The observations $y_1, y_2, \dots, y_n$ are considered as random variables, and the labels m_i of clusters are supposed to be dependent random variables distributed according to the Gibbs field. The vector labeling Gibbs field $\mathbf{x} = \{x_1, x_2, \dots, x_n\}$ is specified on the fully connected directed graph $G = (V, E)$ with the set of vertices $V = \{0, 1, \dots, n\}$ and the set of directed arcs $E = \{(i, j) \mid i, j \in V\}$. It takes values in the space of labels $\mathcal{M}^V$, i.e. $x_i \in \mathcal{M},\ i \in V$, and is either the exponential or the Gaussian form.

It is proven the OBDR are either

$$\widehat{\mathbf{m}}_{exp} = \arg\min_{m \in \mathcal{M}^V} \left\{ \lambda_i \sum_{i \in V} |f_i - m_i| + \sum_{(i,j) \in E} \beta_{i,j} |m_i - m_j| \right\}$$

or

$$\widehat{\mathbf{m}}_{gaus} = \arg\min_{m \in \mathcal{M}^V} \left\{ \sum_{i \in V} \lambda_i (f_i - m_i)^2 + \sum_{(i,j) \in E} \beta_{i,j} (m_i - m_j)^2 \right\},$$

where $\lambda_i, \beta_{i,j} \ge 0$ are the parameters of the Gibbs distributions.

The problem of identification of $\widehat{\mathbf{m}}_{exp}$ and $\widehat{\mathbf{m}}_{gaus}$ is reduced to determination of minimum cuts of the special networks [2,3,4]. To compute them the special fast network minimum cut algorithm is developed [4]. It allows, for instance, fast classification and segmentation of grayscale and color images.

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