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Search for Customers in a Finite Capacity Queueing System with Phase-type Distributions

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Abstract—With the rapid spread of new technologies many efforts have been addressed towards the modelling of telecommunications systems. Since actual statistics shows that queueing systems characterized by Poisson flows of customers with exponentially distributed service times are not good models for multimedia flows, queueing systems with distributions different from traditional ones have to be investigated. In this paper we deal with a single-server queueing system in which the server requires a search for customers allocated in a finite buffer. Queueing systems with the server searching for customers can be used to evaluate performance of telecommunication systems in which the processor must spend a random time to choose the next item to be processed. We assume PH-distributions for the probability distribution functions of interarrival times, service times and search times. The solution of the equilibrium equations of the underlying Markov process is obtained in a matrix-geometric form. Numerical examples are presented and some system performance indices are computed.

1. INTRODUCTION

The continuous development of new technologies, architectures and protocols and their intensive use has lead to a great effort in stochastic modelling of telecommunications systems. Recent studies have shown that customers arrivals are not well modelled by Poisson processes, they are in fact either more bursty or much more deterministic. Packet service times in communications networks are very often not exponentially distributed. Finally, real service stations cannot be schematized using infinite buffer queues, e.g., the intermediate switching nodes have buffer of finite capacity. New discoveries address research towards the analysis of queueing systems with interarrival and service times distributions different from the traditional ones.

In particular systems with the server requiring a search for customers are able to describe server systems as computer systems in which the processor has to spend a random time to individuate the next item to be processed.

In Neuts and Ramalhoto [1] stationary state probabilities for a $M/G/1/\infty$ queueing system with search process for customers are obtained.

In Bocharov *et al.* [2] stationary state probabilities and some performance indices are computed for a queueing model with two Poisson input and the server requiring a priority search for customers.

In this paper we study a queueing system with input flow, service times and search times characterized by PH-distributions.

Our aim is to solve the equilibrium equations for the analyzed queueing system finding a compact matrix-geometric form for the stationary state probabilities and to compute some indices useful to understand the productivity of the system. Finally we report some numerical examples.

In dealing with the system $PH/PH/1/r$ we follow the matrix approach of [3] for a similar queue without search.

2. MODEL DESCRIPTION

We analyze a queueing system with a single server and a storage of finite capacity r $(1 < r < \infty)$. A flow of customers of the same type arrives to the system from an external source. A customer who arrives when the buffer is full leaves the system without making any next attempt to be served again.

We assume that the flow of customers is a renewal process with a phase-type probability distribution function (PDF)

$$A(x) = 1 - \alpha^T e^{\Lambda x}\mathbf{1}, \quad x > 0, \quad \alpha^T\mathbf{1} = 1,$$

having an irreducible PH-representation (α, Λ) of order l. A customer arriving to the system is placed in the buffer of the system even if the server is empty and there he waits until the server searches for him. The duration of a search for customers has PDF $B_0(x)$. If the buffer is empty a new search begins and its duration has the same PDF $B_0(x)$.

The service discipline is FCFS and the customers service times are i.i.d. random variables with PDF $B_1(x)$.

We suppose that the search process and the service one are of phase-type of order $m_n, n = 0, 1$, with PDF

$$B_n(x) = 1 - \beta_n^T e^{M_n x}\mathbf{1}, \quad x > 0, \quad \beta_n^T\mathbf{1} = 1.$$

We can describe the stochastic behaviour of the queueing system, using the probability phase interpretation of the processes of arrival and service, by an homogenous Markov process $\{\eta(t), t \geq 0\}$ on the following set of states:

$$\chi = \{(i, k, n, j), \quad i = \overline{1, l}, \quad k = \overline{0, r}, \quad n = 0, 1, \quad j = \overline{1, m_n}\}.$$

For any time t, $\eta(t) = (i, k, n, j)$ corresponds to the case in which there are k customers in the buffer, the arrival process is in a fictitious phase i and the search process (if $n = 0$) or the service process (if $n = 1$) is in phase j. Under these assumptions the limit probabilities p_{iknj} of the states (i, k, n, j) exist, do not depend on the initial distribution and coincide with stationary probabilities.

Let us denote

$$\lambda = -\Lambda\mathbf{1}, \quad \mu_n = -M_n\mathbf{1},$$
$$\Lambda^* = \Lambda + \lambda\alpha^T, \quad M_0^* = M_0 + \mu_0\beta_{0,}^T$$
$$M = \begin{pmatrix} M_0 & 0 \\ \mu_1\beta_0^T & M_1 \end{pmatrix}, \quad \widehat{M} = \begin{pmatrix} M_0^* & 0 \\ \mu_1\beta_0^T & M_1 \end{pmatrix},$$
$$\mu = -M\mathbf{1}, \quad \beta^T = (\mathbf{0}_{m_0}^T, \beta_1^T),$$

where **1** is a unity vector consisting of ones. Let us denote the vectors of stationary state probabilities

$$\mathbf{p}_{ikn}^T = (p_{ikn1}, p_{ikn2}, \ldots, p_{ikn,m_n}),$$
$$\mathbf{p}_k^T = (\mathbf{p}_{1k0}^T, \mathbf{p}_{1k1}^T, \mathbf{p}_{2k0}^T, \mathbf{p}_{2k1}^T, \ldots, \mathbf{p}_{lk0}^T, \mathbf{p}_{lk1}^T).$$

The stationary distribution of the process $\{\eta(t), t \geq 0\}$ satisfies the following equations:

$$\mathbf{0}^T = \mathbf{p}_0^T(\Lambda \oplus \widehat{M}) + \mathbf{p}_1^T(I \otimes \mu\beta^T), \tag{2.1}$$
$$\mathbf{0}^T = \mathbf{p}_k^T(\Lambda \oplus M) + \mathbf{p}_{k-1}^T(\lambda\alpha^T \otimes I) + \mathbf{p}_{k+1}^T(I \otimes \mu\beta^T), \quad k = \overline{1, r-1}, \tag{2.2}$$
$$\mathbf{0}^T = \mathbf{p}_r^T(\Lambda^* \oplus M) + \mathbf{p}_{r-1}^T(\lambda\alpha^T \otimes I), \tag{2.3}$$

where $A \otimes B$ and $A \oplus B$ are the Kronecker product and sum, respectively, of matrices A and B.

3. STATIONARY PROBABILITIES IN A MATRIX-GEOMETRIC FORM

We prove a theorem which allows to express stationary probabilities of the queueing system under consideration in a matrix-geometric form.

Let us denote

$$\mathbf{p}^T = \mathbf{p}_0^T(I \otimes \mathbf{1}),$$
$$\widetilde{M} = \Lambda \otimes (\mathbf{1}\beta^T - I) - I \otimes M,$$
$$\widetilde{\Lambda} = (\mathbf{1}\alpha^T - I) \otimes M - \Lambda \otimes I, \quad \widehat{\Lambda} = -(\Lambda \oplus \widehat{M}) + \mathbf{1}\alpha^T \otimes \widehat{M},$$
$$W_0 = (\Lambda \otimes \beta^T)(\Lambda \oplus \widehat{M})^{-1}, \quad W_1 = \widehat{\Lambda}\widetilde{M}^{-1},$$
$$W = \widetilde{\Lambda}\widetilde{M}^{-1}, \quad W_r = -(\lambda\alpha^T \otimes I)(\Lambda^* \oplus M)^{-1},$$
$$Z = W_0 W_1 W^{r-2}(W_r\widetilde{M} - \widetilde{\Lambda})(I \otimes \mathbf{1}).$$

Theorem 3.1. *If $\beta_n(-\sigma_i) \neq 0$ for all eigenvalues $\{\sigma_i\}$ of the matrix Λ, where $\beta_n(s)$ is the Laplace-Stieltjes transform of the PDF $B_n(x)$, then the stationary distribution $\{\mathbf{p}_k, k = \overline{0,r}\}$ is represented in the form*

$$\mathbf{p}_k^T = \begin{cases} \mathbf{p}^T W_0, & k = 0, \\ \mathbf{p}^T W_0 W_1 W^{k-1}, & k = \overline{2, r-1}, \\ \mathbf{p}^T W_0 W_1 W^{r-2} W_r, & k = r, \end{cases} \tag{3.1}$$

where the vector $\mathbf{p}$ is found with accuracy up to a constant from the system of equations

$$\mathbf{p}^T Z = \mathbf{0}^T,$$

and the constant is determined from the normalizing condition.

Proof. First, we obtain the equations of the local balance. Multiplying both sides of equations (2.1)–(2.3) on the right for $\mathbf{1} \otimes \mathbf{1}$, we get

$$\mathbf{p}_0^T(\lambda \otimes \mathbf{1}) = \mathbf{p}_1^T(\mathbf{1} \otimes \mu),$$
$$\mathbf{0}^T = -\mathbf{p}_k^T(\lambda \otimes \mathbf{1}) - \mathbf{p}_k^T(\mathbf{1} \otimes \mu) + \mathbf{p}_{k-1}^T(\lambda \otimes \mathbf{1}) + \mathbf{p}_{k+1}^T(\mathbf{1} \otimes \mu), \quad k = \overline{1, r-1},$$
$$\mathbf{p}_r^T(\mathbf{1} \otimes \mu) = \mathbf{p}_{r-1}^T(\lambda \otimes \mathbf{1}).$$

Summing up the resulting equations we have the following relationships:

$$\mathbf{p}_k^T(\lambda \otimes \mathbf{1}) = \mathbf{p}_{k+1}^T(\mathbf{1} \otimes \mu), \quad k = \overline{0, r-1}, \tag{3.2}$$

which are the equations of the local balance of the process under study.

Multiplying equation (2.2) with $k = 1$ on the right for $I \otimes (\mathbf{1}\beta^T - I)$, and using relations (3.2) and (2.1) we obtain

$$\mathbf{p}_1^T\widetilde{M} = \mathbf{p}_0^T[-(\Lambda \oplus \widehat{M}) + \mathbf{1}\alpha^T \otimes \widehat{M}],$$

which leads to

$$\mathbf{p}_1^T\widetilde{M} = \mathbf{p}_0^T\widehat{\Lambda}.$$

Multiplying equation (2.2) on the right for $I \otimes (\mathbf{1}\beta^T - I)$, and taking into account (3.2) we get

$$\mathbf{p}_k^T\widetilde{M} = \mathbf{p}_k^T(I \otimes \mu\beta^T) - \mathbf{p}_{k-1}^T(\mathbf{1}\alpha^T \otimes \mu\beta^T) + \mathbf{p}_{k-1}^T(\lambda\alpha^T \otimes I), \quad k = \overline{2, r-1}.$$

At last multiplying (2.2) on the right for $(\mathbf{1}\alpha^T - I) \otimes I$, we have

$$\mathbf{p}_k^T \widetilde{\Lambda} = \mathbf{p}_{k+1}^T (I \otimes \mu\beta^T) - \mathbf{p}_{k+1}^T (\mathbf{1}\alpha^T \otimes \mu\beta^T) + \mathbf{p}_k^T (\lambda\alpha^T \otimes I), \quad k = \overline{2, r-1}.$$

Hence we get

$$\mathbf{p}_{k-1}^T \widetilde{\Lambda} = \mathbf{p}_k^T \widetilde{M}, \quad k = \overline{2, r-1}.$$

Thus we have obtained the following recurrent expressions

$$\begin{cases} \mathbf{p}_1^T \widetilde{M} = \mathbf{p}_0^T \widehat{\Lambda}, & k = 0, \\ \mathbf{p}_k^T \widetilde{M} = \mathbf{p}_{k-1}^T \widetilde{\Lambda}, & k = \overline{1, r-1}, \\ \mathbf{p}_r^T (\Lambda^* \oplus M) = -\mathbf{p}_{r-1}^T (\lambda\alpha^T \otimes I), & k = r. \end{cases} \tag{3.3}$$

Under above assumptions on PH-distributions $A(x)$, $B_n(x)$, $\widetilde{M}$ is not singular. Therefore from these equations we can compute the stationary state probabilities in function of the unknown vector $\mathbf{p}_0^T$. Using $W_1 = \widehat{\Lambda}\widetilde{M}^{-1}$ we can rewrite the first equation in (3.3) in the form

$$\mathbf{p}_1^T = \mathbf{p}_0^T W_1. \tag{3.4}$$

From the second equation in (3.3), using $W = \widehat{\Lambda}\widetilde{M}^{-1}$, we have

$$\mathbf{p}_k^T = \mathbf{p}_{k-1}^T W, \quad k = \overline{1, r-1},$$

from which it follows that

$$\mathbf{p}_k^T = \mathbf{p}_1^T W^{k-1}, \quad k = \overline{1, r-1}.$$

Substituting the expression of $\mathbf{p}_1^T$ obtained in (3.4), we can express $\mathbf{p}_k^T$ in term of $\mathbf{p}_0^T$

$$\mathbf{p}_k^T = \mathbf{p}_0^T W_1 W^{k-1}, \quad k = \overline{1, r-1}. \tag{3.5}$$

Finally, from (3.3), we have that

$$\mathbf{p}_r^T = -\mathbf{p}_{r-1}^T (\lambda\alpha^T \otimes I)(\Lambda^* \oplus M)^{-1},$$

from which, applying relation (3.5), we can reduce the latter equation to the form

$$\mathbf{p}_r^T = \mathbf{p}_0^T W_1 W^{k-1} W_r.$$

So we have obtained the following expression for $\mathbf{p}_k^T$:

$$\mathbf{p}_k^T = \begin{cases} \mathbf{p}_0^T W_1 W^{k-1}, & k = \overline{1, r-1}, \\ \mathbf{p}_0^T W_1 W^{r-1} W_r, & k = r. \end{cases} \tag{3.6}$$

Now we get an expression for $\mathbf{p}_0$. It is easy to show that the following properties hold:

$$(\lambda\alpha^T \otimes I) = \widetilde{\Lambda}(\mathbf{1}\alpha^T \otimes I), \tag{3.7}$$

$$(I \otimes \mu\beta^T) = \widetilde{M}(I \otimes \mathbf{1}\beta^T). \tag{3.8}$$

From (2.1) we have

$$\mathbf{p}_0^T = -\mathbf{p}_1^T (I \otimes \mu\beta^T)(\Lambda \oplus \widehat{M})^{-1}.$$

Taking into account (3.3), the property (3.8) and the definition of $\widehat{\Lambda}$ we obtain

$$\mathbf{p}_0^T = \mathbf{p}_0^T(I \otimes \mathbf{1})(\Lambda \otimes \beta^T)(\Lambda \oplus \widehat{M})^{-1} = \mathbf{p}^T W_0.$$

Therefore we can now rewrite the stationary probabilities (3.6) in the matrix form (3.1) in function of the unknown vector $\mathbf{p}^T$.

It should be noted that transformations reducing the equilibrium equations (2.1)–(2.3) to (3.3) are not equivalent, therefore, in order to be sure that expressions (3.6) define the solution of the equilibrium equations, we have to substitute (3.6) into (2.1)–(2.3).

It is easy to prove that substituting (3.6) into equations (2.1), (2.3) we turn them into an identity. It remains to verify that also (2.2) is identically satisfied.

Let us consider equation (2.2) for a fixed $k = 1, 2, \ldots, r-2$. Using properties (3.7), (3.8) and relations (3.3) we have

$$\mathbf{0}^T = \mathbf{p}_k^T[\Lambda \oplus M + \widetilde{M}(\mathbf{1}\alpha^T \otimes I) + \widetilde{\Lambda}(I \otimes \mathbf{1}\beta^T)] = \mathbf{p}_k^T(\Lambda \oplus M)H, \quad k = \overline{1, r-1},$$

where

$$H = I \otimes I - \mathbf{1}\alpha^T \otimes I - I \otimes \mathbf{1}\beta^T - \mathbf{1}\alpha^T \otimes \mathbf{1}\beta^T.$$

Introducing the notation

$$F = (\Lambda \oplus M)H,$$

we can rewrite the last relation in the following form:

$$\mathbf{p}_k^T F = \mathbf{0}^T, \quad k = \overline{1, r-2}.$$

It easy to prove that

$$F = -\widetilde{M}H = -\widetilde{\Lambda}H, \tag{3.9}$$

from which it follows that

$$\widetilde{M}H = \widetilde{\Lambda}H. \tag{3.10}$$

Applying (3.9), (3.10) and (3.3) we obtain

$$\mathbf{0}^T = \mathbf{p}_k^T F = -\mathbf{p}_k^T \widetilde{M}H = -\mathbf{p}_{k-1}^T \widetilde{\Lambda}H = \mathbf{p}_{k-1}^T F.$$

For induction we have

$$\mathbf{0}^T = \mathbf{p}_k^T F = \mathbf{p}_{k-1}^T F = \ldots = \mathbf{p}_1^T F.$$

It is easy to verify that

$$(I \otimes \beta^T)H \equiv \mathbf{0}^T, \quad (\alpha^T \otimes I)H \equiv \mathbf{0}^T.$$

Taking into account equation (2.2) and properties (3.7), (3.8) we obtain

$$\mathbf{p}_k^T F = \mathbf{p}_k^T(\Lambda \oplus M)H =$$
$$= -\mathbf{p}_{k-1}^T \widetilde{\Lambda}(\mathbf{1} \otimes I)(\alpha^T \otimes I)H - \mathbf{p}_{k+1}^T \widetilde{M}(\mathbf{1} \otimes I)(I \otimes \beta^T)H \equiv \mathbf{0}^T, \quad k = \overline{1, r-2}.$$

Thus the substitution of the solution (3.1) into equations (2.1), (2.3) for $k = \overline{1, r-2}$, and (3.3) leads to identities. Now we shall substitute the solution (3.1) into equation (2.3) for $k = r-1$.

Then using the relations (3.3), (3.6) and the properties (3.7), (3.8), we get

$$\mathbf{0}^T = \mathbf{p}_{r-1}^T \Lambda \oplus M + \mathbf{p}_{r-1}^T \widetilde{M}\widetilde{\Lambda}^{-1}(\lambda\alpha^T \otimes I) + \mathbf{p}_r^T (I \otimes \mu\beta^T) =$$
$$= \mathbf{p}^T W_0 W_1 W^{r-2}[\Lambda \oplus M + \widetilde{M}(\mathbf{1}\alpha^T \otimes I) + W_R \widetilde{M}(I \otimes \mathbf{1}\beta^T)].$$

Hence using some transformations we can express the last relation in the following form:

$$\mathbf{p}^T W_0 W_1 W^{r-2}(W_R \widetilde{M} - \widetilde{\Lambda})(I \otimes \mathbf{1})(I \otimes \beta^T) = \mathbf{0}^T,$$

and we obtain

$$\mathbf{p}^T Z = \mathbf{0}^T.$$

Owing on the fact that the system of equilibrium equations has a unique solution within to a constant we can formulate also the same assertion for the last system of equations. Thus, the Theorem 3.1 is proved.

4. PERFORMANCE INDICES

Let us introduce some useful performances indices. If we denote the arrival and the service rates by λ and μ_1, respectively, we have

$$\lambda^{-1} = -\alpha^T \Lambda^{-1}\mathbf{1}, \quad \mu_1^{-1} = -\beta^T M_1^{-1}\mathbf{1}.$$

It is easy to prove another expression for λ

$$\lambda = \sum_{k=0}^{r} \mathbf{p}_k^T (\lambda \otimes \mathbf{1}). \tag{4.1}$$

Let us denote $\rho = \lambda/\mu_1$ the system traffic rate, λ_D the throughput defined as the average number of customers served by the system, per unit of time, also known as the departure rate and λ_A the rate of the customers flow actually accepted by the system. The following relations hold:

$$\lambda_A = \sum_{k=0}^{r-1} \mathbf{p}_k^T (\lambda \otimes \mathbf{1}), \tag{4.2}$$

$$\lambda_D = \sum_{k=0}^{r} \mathbf{p}_k^T (\mathbf{1} \otimes \mu_{11}^T), \tag{4.3}$$

where $\mu_{11}^T = (\mathbf{0}_{m_0}^T, \mu_1^T)$.

The quantities

$$p_0 = \sum_{k=0}^{r} \mathbf{p}_k^T (\mathbf{1} \otimes (\mathbf{1}^T, \mathbf{0}_{m_1}^T)), \quad u = 1 - p_0,$$

determine, respectively, the probability that the server is idle and the utilization factor of the service facility. The mean number of customers in the buffer is given by

$$N_B = \sum_{k=0}^{r} k(\mathbf{p}_k^T \mathbf{1}_{l(m_0+m_1)}). \tag{4.4}$$

Finally we define the loss probability of a customer arriving to the system. In the stationary state of the system the loss probability, p_L, is given by the formula

$$p_L = 1 - \frac{\lambda_D}{\lambda}. \tag{4.5}$$

Relations (4.1) and (4.5) lead to another expression for p_L

$$p_L = \frac{1}{\lambda}\mathbf{p}_r^T(\lambda \otimes \mathbf{1}). \tag{4.6}$$

Since the Markov process describing the behaviour of the queueing system is ergodic it is easy to prove that

$$\lambda_D = \mu(1 - p_0). \tag{4.7}$$

From (4.5) and (4.7) it follows the obvious relationship

$$\rho(1 - p_L) = 1 - p_0. \tag{4.8}$$

The left-hand side of the previous relation indicates the value of the traffic rate actually accepted by the system. This equality has an obvious physical meaning.

5. NUMERICAL EXAMPLES

We have implemented all the formulas in the Theorem 3.1 in the software Mathematica^{TM} 4.0. Let be

$$P_{0,k} = \mathbf{p}_k^T(I \otimes (\mathbf{1}_{m_0}, \mathbf{0}_{m_1}^T)), \quad k = \overline{0, r},$$
$$P_{1,k} = \mathbf{p}_k^T(I \otimes (\mathbf{0}_{m_0}^T, \mathbf{1}_{m_1}^T)), \quad k = \overline{0, r},$$
$$P_k = \mathbf{p}_k^T\mathbf{1}, \quad k = \overline{0, r},$$

where $P_{i,k},\ k = \overline{0, r}$ is the number of customers in the system when it is in search phase (if $i = 0$) or it is busy (if $i = 1$). Then $P_k = P_{0,k} + P_{1,k}$ is the queue length distribution.

We have calculated some numerical examples.

Example 1. We consider $r = 30$,

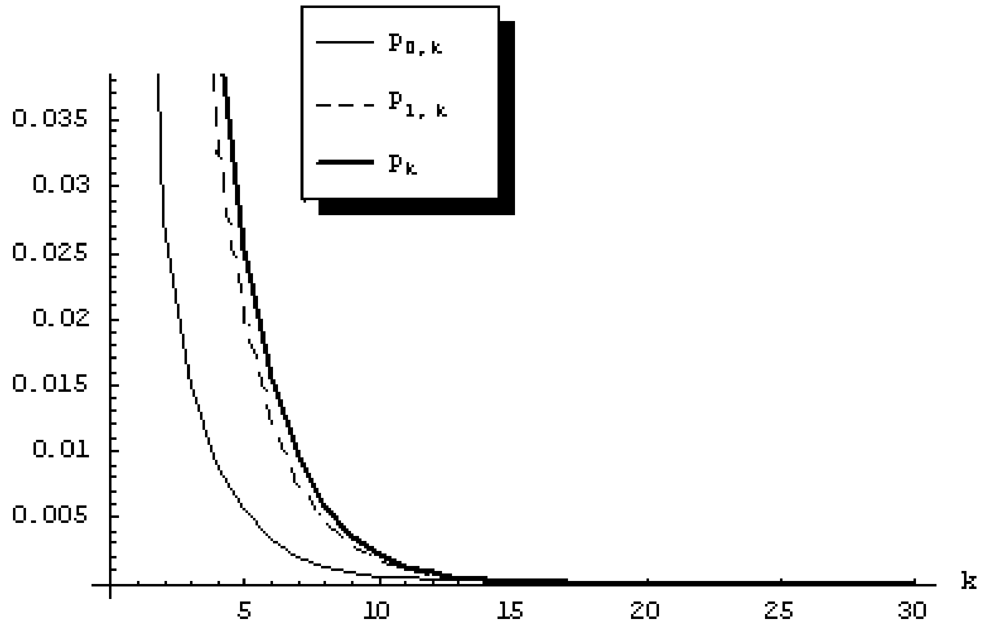
$$\Lambda = \begin{pmatrix} -0.4 & 0.4 \\ 0 & -0.4 \end{pmatrix}, \quad \alpha^T = (0.2, 0.8),$$
$$M_0 = \begin{pmatrix} -2 & 0 \\ 0 & -3 \end{pmatrix}, \quad \beta_0^T = (0.4, 0.6),$$
$$M_1 = \begin{pmatrix} -0.8 & 0.8 \\ 0 & -0.8 \end{pmatrix}, \quad \beta_1^T = (0.3, 0.7).$$

In this case the system traffic rate ρ is equal to 0.542.

In Table 1 we report values of p_o p_L, and N_B for different size of the buffer capacity.

Table 1

$\rho = 0.542$	$r = 5$	$r = 10$	$r = 15$	$r = 20$	$r = 25$	$r = 30$
p_0	0.366078	0.334905	0.326759	0.3242	0.323351	0.323065
p_L	0.0637457	0.0177053	0.00567546	0.00189565	0.000641621	0.000219421
N_B	1.59185	2.65742	3.23229	3.51193	3.63822	3.69215



Example 2. In this example we use for $r, M_0, M_1, \alpha, \beta_0, \beta_1$ the same values as in the previous example and for Λ.

$$\Lambda = \begin{pmatrix} -0.6 & 0.6 \\ 0 & -0.6 \end{pmatrix}.$$

The system traffic rate ρ is equal to 0.8125.

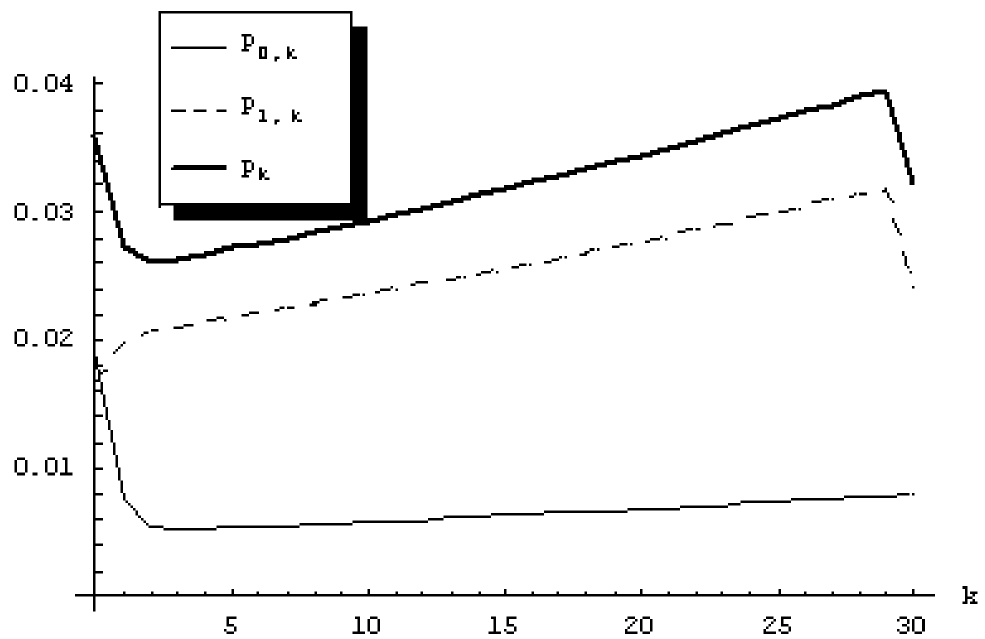


Table 2

$\rho = 0.875$	$r = 5$	$r = 10$	$r = 15$	$r = 20$	$r = 25$	$r = 30$
p_0	0.292593	0.248424	0.231389	0.222382	0.216824	0.213063
p_L	0.129345	0.074984	0.0540173	0.0429314	0.0360911	0.0314647
N_B	2.27522	4.85069	7.51758	10.2578	13.0668	15.9426

To check the computation we have used the relation (4.8).

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