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# Continuous Convex Optimization Algorithms and Ideal Sliding Mode

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**Abstract**—The characteristics of the sliding mode based on continuous convex programming algorithms were discussed. The ideal sliding mode was shown to occur in the absence of infinite number of switching.

## 1. INTRODUCTION

Let us consider a general formulation of convex programming problem, namely:

$$f(x) \to \min,$$

$$x \in G = \{x \in R^n : g_i(x) \le 0, i = 1, ..., m\}, \tag{1}$$

where all functions $f, g_i, i = 1, ...m$, are the proper convex functions that are continuously differentiable on $R^n$.

To solve the problem (1), we shall use the method of exact penalty functions, and replace (1) by the problem of determining the unconditional minimum of nonsmooth convex function

$$F(x, q) = f(x) + q \sum\nolimits_{i=1}^{m} g_i^+(x), \tag{2}$$

where

$$g_i^+(x) = \begin{cases} 0, & g_i(x) \le 0 \\ g_i(x), & g_i(x) > 0, \end{cases} \qquad i = 1, ..., m,$$

and $q > 0$ is the penalty coefficient. The existence conditions for the critical penalty coefficient $q_0 < \infty$ that makes problem (1) and (2) equivalent for all $q > q_0$ are well known [1].

To determine the unconditional minimum of the function (2), one can employ the continuous subgradient algorithm [2] and consider the system of differential inclusions

$$\frac{dx}{dt} \in -\partial_x F(x, q), x(t_0) = x^0 \in R^n, \tag{3}$$

where $\partial_x F(x, q)$ is the partial subdifferential of the function (2) that is convex in the argument $x \in R^n$. In the case under consideration $\partial_x F(x, q)$ is a convex bounded closed set for all $x \in R^n$ and $q > 0$, and $\partial_x F(x, q)$ is the maximal monotone operator.

It is known [3] that for any $x(t_0) = x^0 \in R^n$ there exists a unique solution $x(t)$ of (3) with maximal monotone operator $\partial_x F(x, q)$, and this solution $x(t)$ defined for all $t \ge t_0$.

## 2. FINITE CONVERGENCE IN CONTINUOUS OPTIMIZATION ALGORITHMS AND IDEAL SLIDING MODE

The approach to the problems of convex optimization as based on the system of differential inclusions (3) is in agreement with the sliding mode-based method of solving the optimization problems [4].

The conditions formulated in [4] as those for existence of sliding modes arise in fact on some surfaces $g_i(x) = 0$, $i = 1, ..., m$, and/or their intersections.

At the same time, the solutions of (3) have some interesting characteristics allowing one to assert that they correspond to the ideal sliding mode.

The present author introduced the notion of **sharp minimum** [2]. For the case at hand, function (3) will be said to have the sharp minimum if the following condition is satisfied:

$$\inf_{x \in R^n / X^*} \min_{y \in \partial_x F(x,y)} \left\langle y, \partial_x F^0(x,q) \right\rangle \geq \nu > 0, \tag{4}$$

where $\partial_x F^0(x,q)$ is the canonical restriction [3] of the multivalued operator $\partial_x F(x,q)$, and $X^*$ is the set of the minimum points of (2) which, under an appropriate choice of the penalty coefficient $q$ coincides with the set of the optimal solutions of the original problem (1); we assume here without loss of generality that the existence conditions for the critical penalty coefficient are met.

As follows from the theorem of [2], any solution $x(t)$ of the system of differential inclusions (3) reaches the points of the set $X^*$ in a finite time depending on the choice of initial approximation $x^0$ and $\nu$ in (4).

Therefore, beginning from some finite time instant $T^*(x^0, \nu)$, the solution $x(t)$ belongs to the set $X^*$ where the right side of system (3) is identically zero.

Since the solution $x(t)$ is an absolutely continuous function, the function $x(t)$ has the derivative—and not only the right derivative—almost everywhere. This assertion suggests that the motion in system (3) indeed takes place on the corresponding surfaces or their intersections, that is, is the ideal sliding mode. Therefore, the ideal sliding modes exist (we do not discuss here technical feasibility of such systems), and motion in such modes does not consist of an infinite number of switching.

## REFERENCES

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