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Multicriteria Steiner Tree Problem for Communication Network

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Abstract—This paper addresses combinatorial optimization schemes for solving the multicriteria Steiner tree problem for communication network topology design (e.g., wireless mesh network). The solving scheme is based on several models: multicriteria ranking, clustering, minimum spanning tree, and minimum Steiner tree problem. An illustrative numerical example corresponds to designing a covering long-distance Wi-Fi network (static Ad-Hoc network). The set of criteria (i.e., objective functions) involves the following: total cost, total edge length, overall throughput (capacity), and estimate of QoS. Obtained computing results show the suggested solving scheme provides good network topologies which can be compared with minimum spanning trees.

1. INTRODUCTION

Many years minimum spanning tree problem (MST) is a basic scientific/telecommunication problem for design of communication/access/computer network topology, for network routing ([3], [4], [5], [6], [17], [18], [21]). In this problem a traditional objective function consists in minimum total length (or weight) of spanning tree edges. Minimum Steiner tree problem (STP) can provide decreasing of the above-mentioned total length by usage of additional nodes (vertices) ([4], [7], [8], [17], [21], [23]). This problem is studied in mathematics (e.g., [2], [7], [8]), in combinatorial optimization (e.g., [4], [21], [23]). In recent two decades the significance of STP was increased (e.g., active usage in communication networks for topology design, routing, protocol engineering, etc.) (e.g., [17], [21]). STP belongs to class of NP-hard problems (e.g., [4]) and exact enumerative solving methods (e.g., [23]) or approximation algorithms (e.g., heuristics) (e.g., [20]) are used.

A brief survey of well-known kinds of Steiner tree problems is presented in [14]. STP with two criteria was investigated in [22]. A description of using a partitioning-synthesis heuristic based on Hierarchical Morphological Multicriteria Design approach for Steiner tree problem was described in [10]. In this article multicriteria Steiner tree problem (multicriteria STP) is firstly suggested. A static Ad-Hoc communication network is examined as an application domain. In the multicriteria spanning problem, each edge has the following attributes: (i) length (cost), (ii) throughput (capacity), (iii) reliability or QoS parameters. Our composite (four-stage) solving scheme is targeted to building some Pareto-effective Steiner trees (i.e., alternative solutions). The solving scheme consists of stages: (a) building a spanning tree, (b) clustering of network nodes (by a modification of agglomerative algorithm), (c) building a Steiner tree for each obtained node cluster, and (d) revelation of Pareto-effective solutions and their analysis.

Presented numerical examples for a wireless communication network illustrate the suggested design approach. Computing was based on authors MatLab programs (http://www.mathworks.com/). A preliminary material was presented as a conference paper [15].

2. SPANNING PROBLEMS

A static multihop wireless network is under examination. Multihop node paths going through a set of nodes are used for two-node communications. Altitude map is introduced and four main criteria are assigned for each P2P connection. The altitude is the one of significant parameters because it affects not only network productivity (wireless links require line-of-sight clearance) but also link costs. The examined network is considered as undirected graph $G(V, E)$ where V is the set of nodes (vertices) and E is the set of edges. It is assumed that the network is two dimensional one, though node stations are at different height. This fact has an affect on altitude criterion for each P2P connection.

2.1. Parameters

The parameters under consideration are following:

1. A distance between two vertices $v_i, v_j \in V$: $l_{i,j}$.
2. QoS characteristics for two vertices $v_i, v_j \in V$: q_{ij}.
3. An altitude between two vertices $v_i, v_j \in V$: δ_{ij}.
4. A cost of connection between $v_i, v_j \in V$ depends on δ_{ij} and q_{ij}: $c_{ij} = F(\delta_{ij}, q_{ij})$. Here it is assumed that F is proportional to linear aggregate δ_{ij}^3 and q_{ij}.

2.2. Basic engineering problem

A set of transmission stations is considered in the network that is represented by graph G. Each connection/edge of G is evaluated upon four characteristics above. The examined problem is:

Find Pareto-effective Steiner tree for graph G while taking into account the following criteria:
(i) overall cost: $C_{st} = \sum_{e_{ij} \subset G_{st}} c_{ij}$;
(ii) total network length: $L_{st} = \sum_{e_{ij} \subset G_{st}} l_{ij}$;
(iii) overall QoS: $Q_{st} = \sum_{e_{ij} \subset G_{st}} q_{ij}$; *and*
(iv) summarized altitude: $\Delta_{st} = \sum_{e_{ij} \subset G_{st}} \delta_{ij}$
where $G_{st}(V', E')$ *is a Steiner tree,* $e_{ij} \subset E'$: $V' \supseteq V$ *and* E' *is the extended set of edges.*

2.3. Problem formulations

The basic problem (Minimum Spanning Tree MST) is:

$$min \sum_{e_{ij} \subset G_{span}} l_{ij}$$

where

$$G_{span}(V', E') : \; V' \equiv V, \; E' \equiv E.$$

Here there are well-known solving methods such as Prim's algorithm and Kruskal's algorithm (e.g., [1]).

If extra vertices can be added to minimize overall length, Steiner tree problem (STP) can be examined:

$$min \sum_{e_{ij} \subset G_{st}} l_{ij}$$

where

$$G_{st}(V', E') : \ V' \supseteq V.$$

This problem is NP-hard. There are many heuristics proposed during the recent years (e.g., [7], [20]).

An extension of MST problem is targeted to finding an efficient set of overall characteristics (i.e., optimization by vector function) (multicriteria MST, i.e., MMST):

$$min \ C_{span}(G_{span}), \ min \ L_{span}(G_{span}), \ max \ Q_{span}(G_{span}), \ min \ \Delta_{span}(G_{span})$$

where:

$$G_{span}(V', E') : \ V' \equiv V, \ E' \equiv E$$

Here a possible solving method consists in multicriteria ranking of graph edges (by estimates) and using standard approaches for MST.

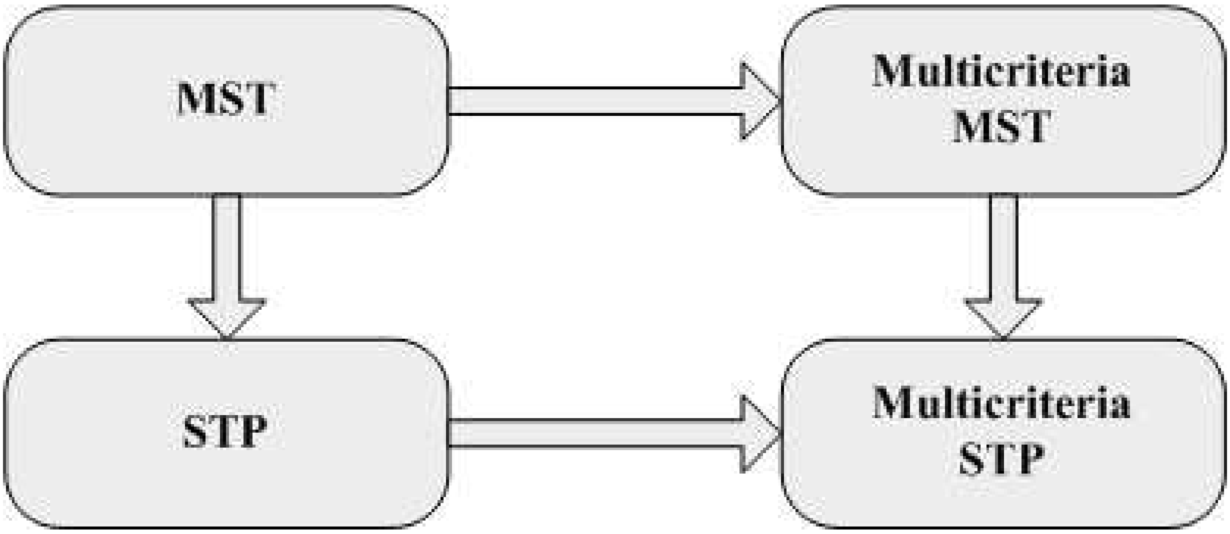

Figure 1. Interconnection of problem formulations

Multicriteria STP (MSTP) is an extension of STP. Here optimization is based on vector function:

$$min \ C_{st}(G_{st}), \ min \ L_{st}(G_{st}), \ max \ Q_{st}(G_{st}), \ min \ \Delta_{st}(G_{st})$$

where:

$$G_{st}(V', E') : \ V' \supseteq V$$

Figure 1 depicts interconnection for the problems above.

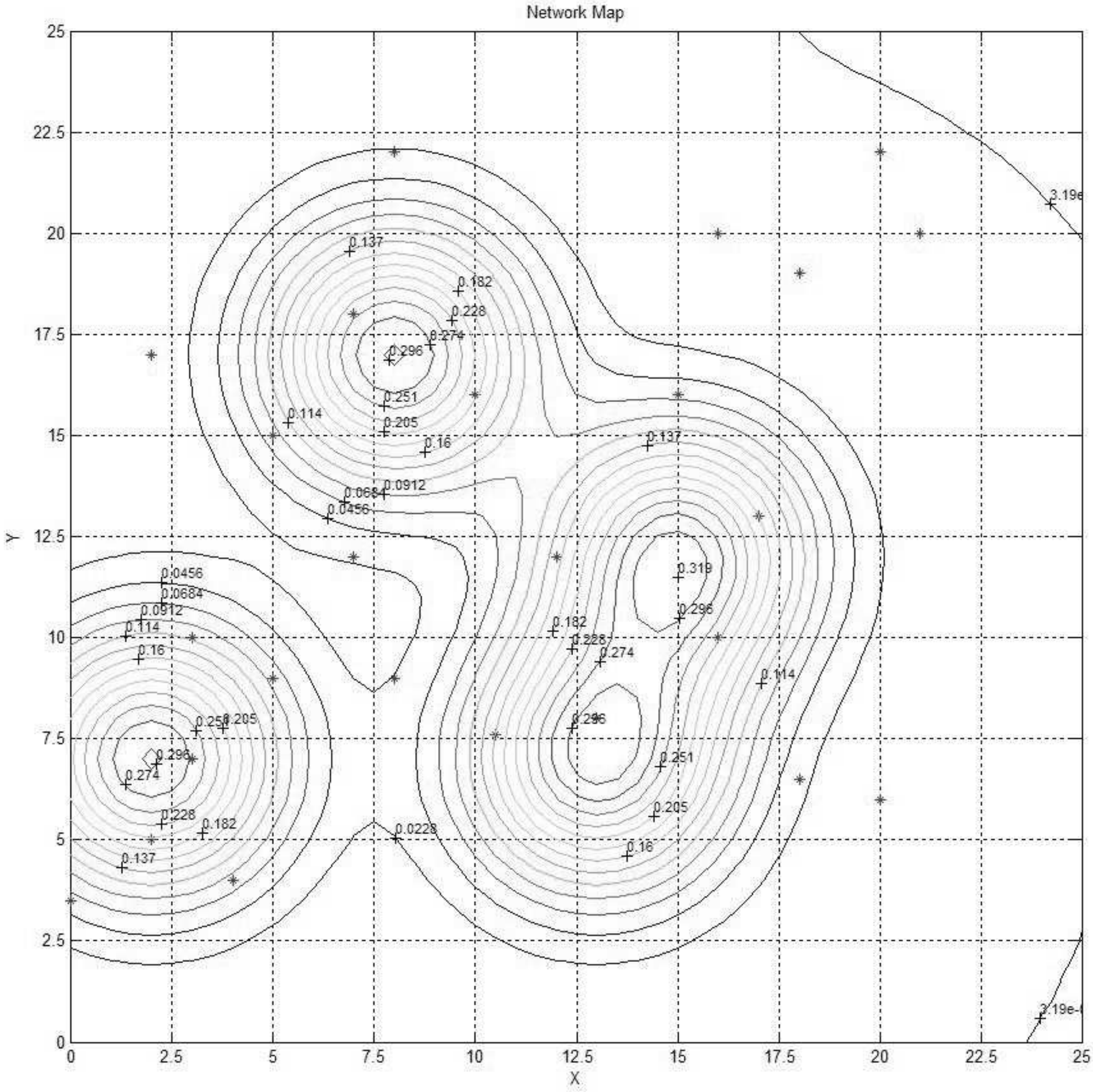

Figure 2. Initial communication network (example)

3. SOLVING SCHEME, EXAMPLE

Our four-stage composite solving scheme (macro-heuristic) is:

Stage 1. Building a multicriteria spanning tree for the initial network (a modified Prim's algorithm [1]).

Stage 2. Clustering of network nodes (by a modification of agglomerative algorithm [12]).

Stage 3. Building a spanning Steiner tree for each obtained node cluster (a modified Melzak's algorithm [7]).

Stage 4. Revelation of Pareto-effective solutions and their analysis.

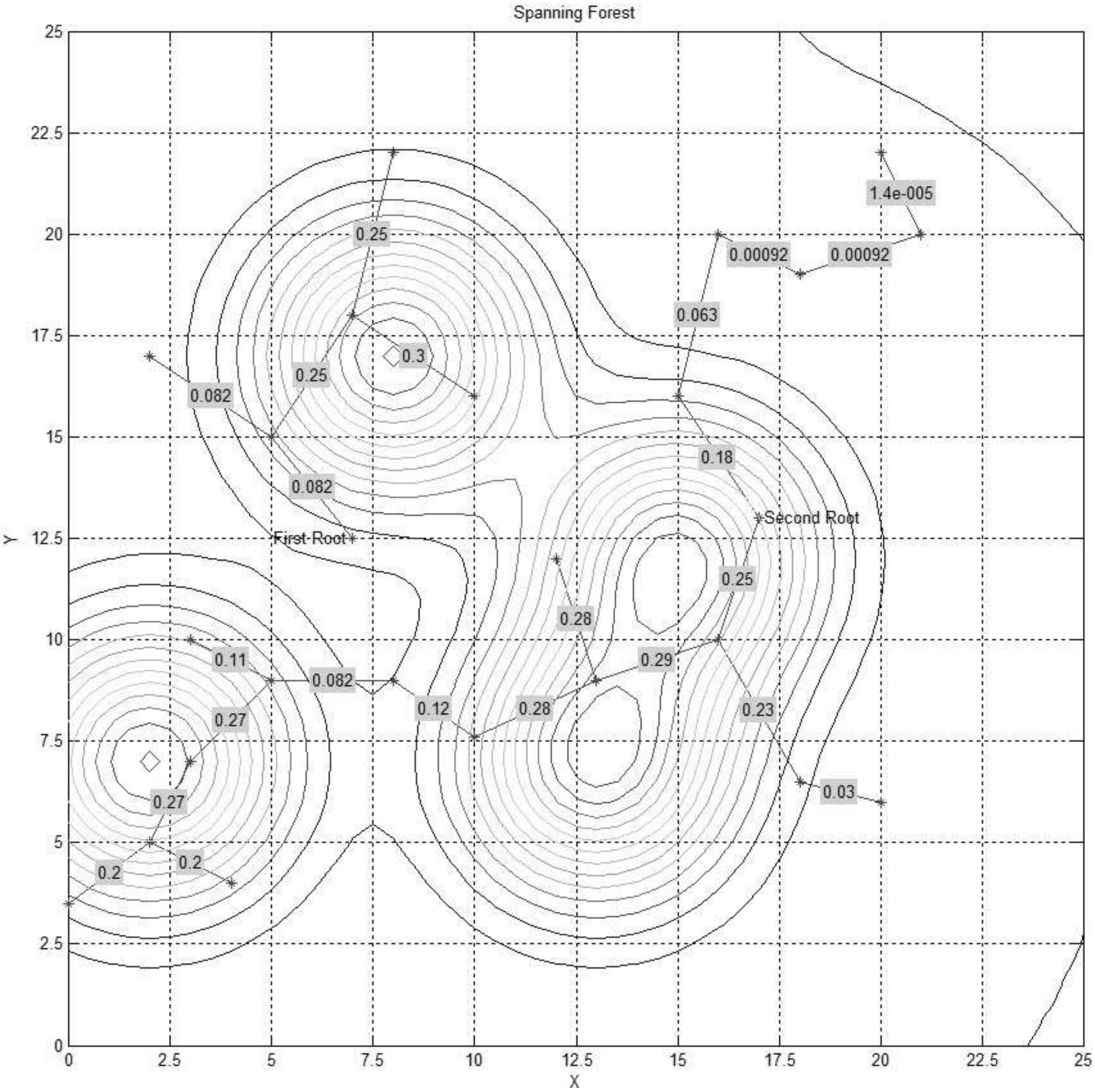

Figure 3. Spanning tree (example)

Let us consider our solving scheme. Concurrently, our numerical example is described. Figure 2 depicts the initial communication network. Thus stages of the solving schemes are considered.

Stage 1 (building a multicriteria spanning tree) is based in the following initial data: the set of network elements (graph vertices, access points). Here a modified Prim's algorithm is used [1]: addition to an existing built tree a "most close subtree"(or a vertex). Multicriteria ranking is based on quadratic utility function. At the first step an initial set of roots is selected and, as a result, "spanning forest"is obtained. At the end step several vertices are selected to extend the solution set. A solution of minimum spanning tree (MST) is depicted in Figure 3. Figure 4 depicts the solution for multicriteria spanning tree MST.

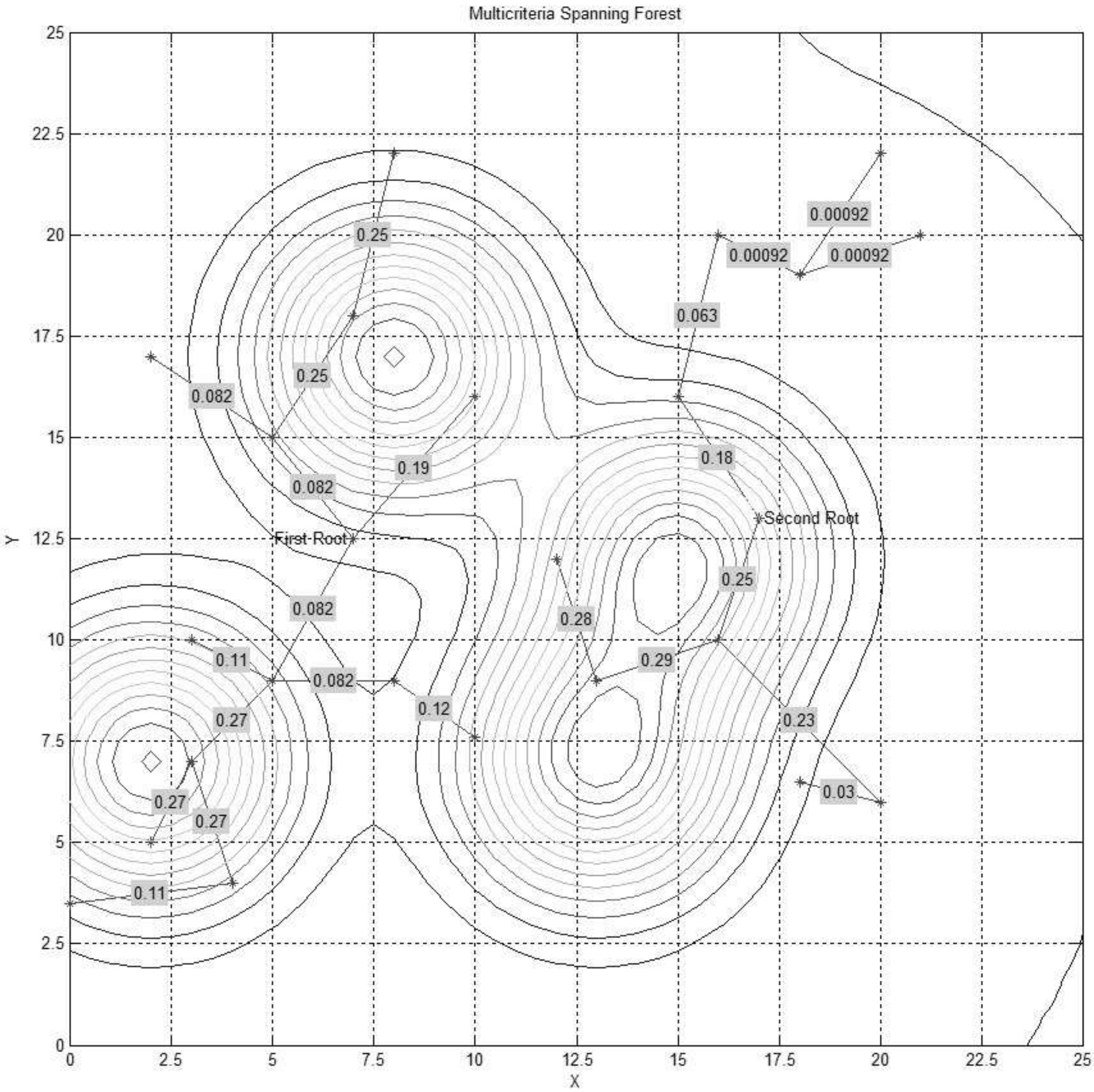

Figure 4. Multicriteria spanning tree (example)

At the stage of clustering (Stage 2), clusters as groups of close vertices in the spanning structure are defined. Here a cluster includes about 5...6 vertices, this cardinality of a cluster elements set is useful to decrease complexity of the solving process for Steiner tree problem (Stage 3). The modified agglomerative algorithm for hierarchical clustering is used [12]. The result of clustering is depicted in Figure 5.

A modified Melzak's algorithm [7] (decreased complexity) is used for building spanning Steiner tree (MSTP) for each cluster (Stage 3). The resultant spanning Steiner tree is depicted in Figure 6.

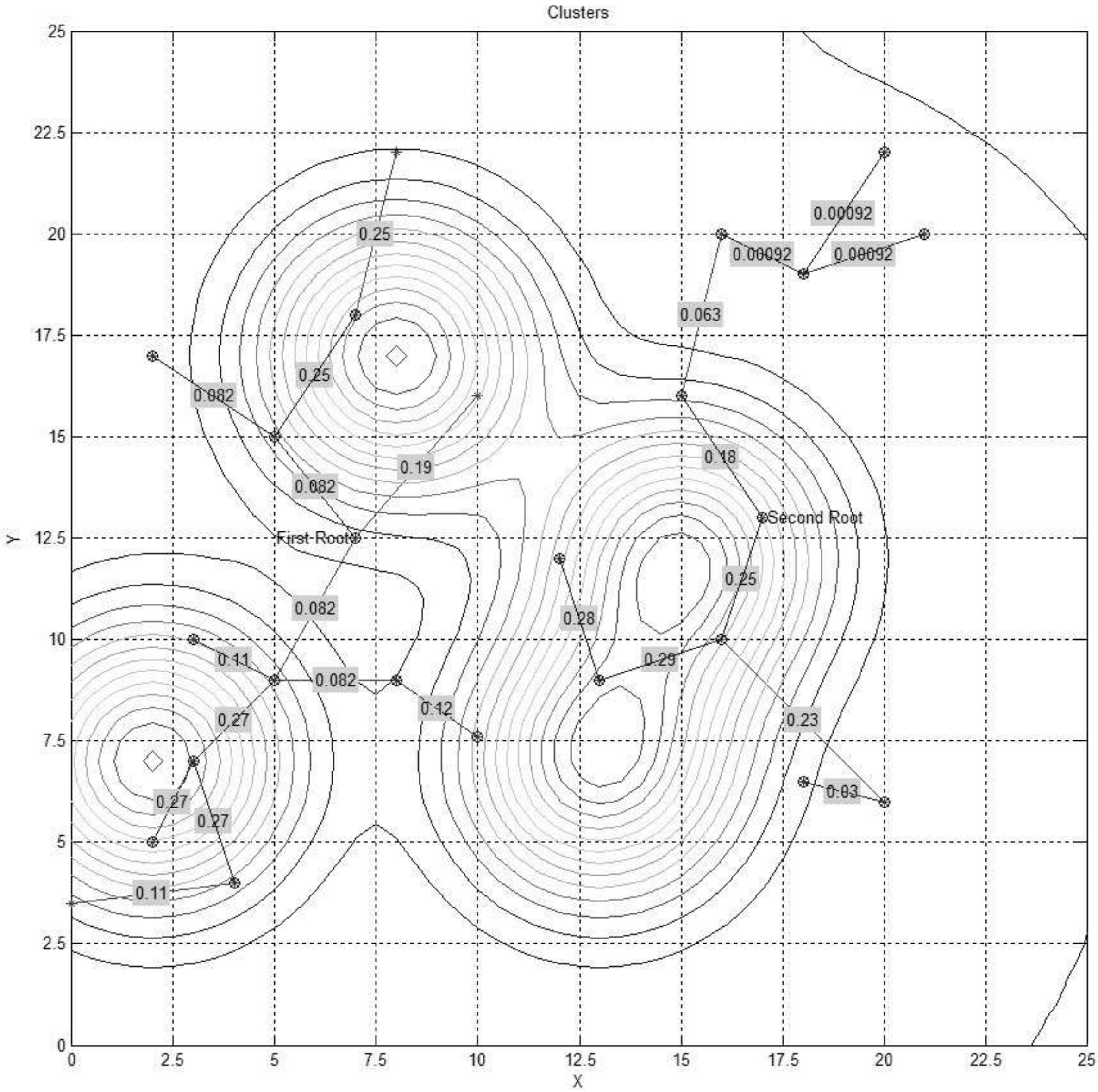

Figure 5. Clustering (example)

At the Stage 4 revelation of Pareto-effective spanning structures is executed. Four above-mentioned objective functions are used. Figure 7 illustrates six Pareto-effective multicriteria spanning Steiner trees (network topology solutions).

Basic algorithms for topology design are shown in Table 1.

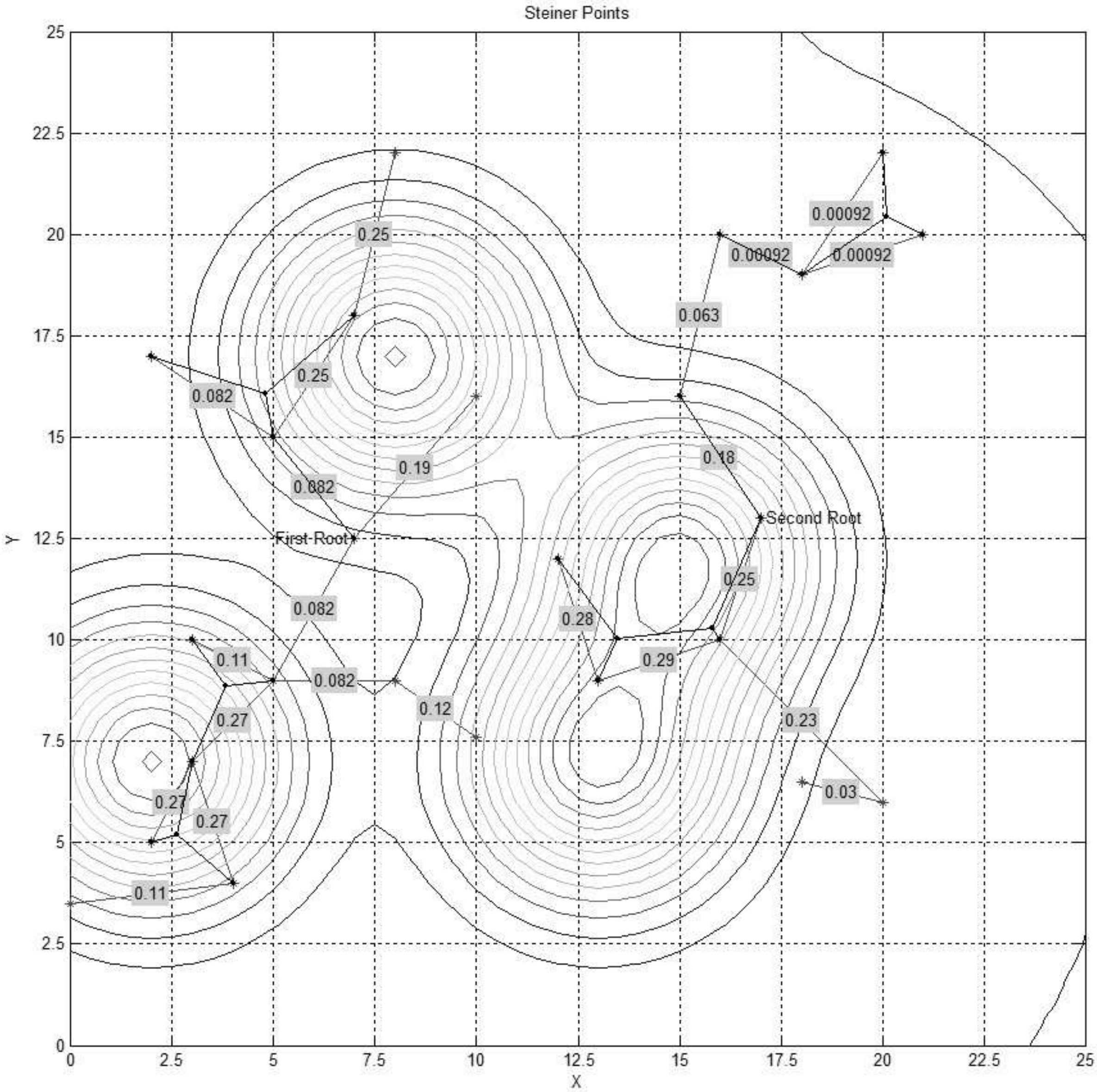

Figure 6. Steiner tree (example)

Table 1. Basic approaches for network topology design

Topology design problems	Basic algorithms	Used algorithms
1.Minimum spanning tree problem MST	1.Kruskal's algorithm [1] 2.Prim's algorithm [1]	Prim's algorithm [1]
2.Steiner tree problem STP	1.Melzak's algorithm [7] 2.Winter's algorithm [7] 3.Simulated annealing [16] 4.Evolutionary methods [9]	Melzak's algorithm [7]
3.Mulicriteria spanning tree MMST	1.Weighted sum function [19] 2.Maximin method [19]	Weighted sum function [19]
4.Mulicriteria Steiner tree MSTP		Methods above

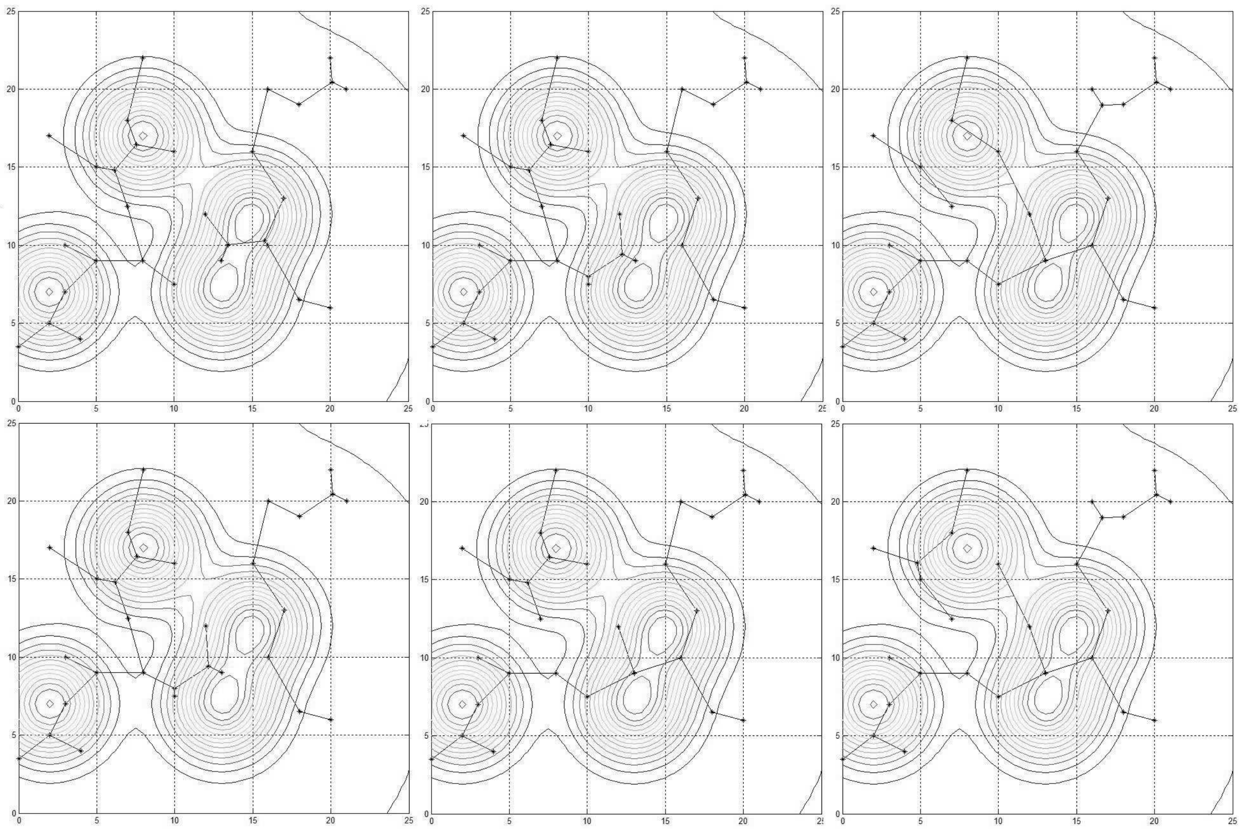

Figure 7. Pareto-effective solutions for Steiner tree problem

4. COMPARISON OF APPROACHES

Table 2 integrates computing results of multicriteria comparison of three approaches: (a) minimum spanning tree problem (MST, Figure 3), (b) multicriteria spanning tree problem (MMST, Figure 4), and (c) multicriteria Steiner tree problem (MSTP, Figure 7). For the comparison, total estimates for four considered objective functions are computed for each approach results and revelation of Pareto-effective solutions (i.e., estimates vectors) is executed: two series steps with selection of the 1st Pareto-effective solution set and the 2nd Pareto-effective solution set (after deletion of the 1st Pareto-effective solution set).

5. CONCLUSION

In the paper, multicriteria spanning Steiner tree problem is firstly suggested for communication network (a case of wireless communication network). The solving scheme (a composite macro-heuristic) consists of the following stages: (i) spanning an initial network by a spanning tree, (ii) clustering of network nodes, (iii) building of spanning Steiner tree for each obtained cluster (a subnetwork), and (iv) revelation and analysis of alternative spanning Pareto-effective Steiner trees. Evidently, it can be reasonable to examine modifications of the used solving scheme and its stages, for example: (a) improvement of clustering methods, (b) increasing of clusters (i.e., cluster node sets), (c) usage and comparison of various algorithms for Steiner tree problem. In addition, a special research computing experiments may be carried out to study the suggested solving scheme, a modified its versions, and evolutionary optimization heuristics.

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Table 2. Comparison of network topology design approaches

Network topology design approach	Length L	Cost C	Altitude Δ	QoS Q	Pareto layer
1.Minimal spanning tree MST (Figure 3)	69.98	3.83	222.50	43.56	2
2.Mulicriteria spanning tree MMST (Figure 4)	76.35	3.45	265.75	41.26	1
3.Mulicriteria Steiner tree MSTP (Figure 7)	69.19	4.29	202.09	43.56	2
	69.24	4.56	195.92	44.38	1
	69.5	4.17	197.26	44.0	2
	70.16	3.77	217.26	42.13	1
	69.5	4.17	197.26	43.76	1
	69.19	4.29	202.09	42.31	1
	69.24	4.57	195.92	43.13	1
	69.92	3.83	209.76	42.4	1

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