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Trajectory Decision Making Framework

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Abstract—The paper addresses a general view to trajectory (route) decision making framework (i.e., designing a trajectory). The view is based on four-part morphological scheme: (a) routing combinatorial problems (e.g., shortest path problem, minimum spanning tree problems, TSP), (b) assessment scales (i.e., quantitative, ordinal, poset-like), (c) graph/network based models as "solving space" (e.g., k-partite graph), and (d) node/vertex models/types. The following issues are considered: (i) structuring the "design/solving space", (ii) problem statement, (iii) heuristics. A realistic university student trajectory problem (route from BS degree to PostDoc position) is examined.

Key words: decision making, combinatorial optimization, routing, trajectory design, heuristic, student trajectory

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1. INTRODUCTION

In recent decades, the role of dynamic decision making has been increased (e.g., [12, 32, 37]). Planning and scheduling problems play central roles in many domains, for example: (a) motion planning, navigation (e.g., [6, 15, 41, 83]), (b) digital system design (VLSI design) (e.g., [88]), (c) network routing (e.g., [14, 23, 50]), (d) information retrieval (e.g., [9]), (e) scenario-planning (e.g., [11,90]), (f) path design in biomedicine (e.g., [53,58–60]). Note, special advanced integrated planning and scheduling systems (APS) are widely used in manufacturing and supply management (e.g., [25, 65, 80]). Petri nets based approaches are often used for workflows modeling (e.g., in manufacturing systems) (e.g., [69, 96]). Multi-stage decision making systems for complicated problems have been discussed in [84]. In [86], reflexive game theory has been proposed for modeling of multistage decision processes. Many methods based on artificial intelligence have been developed for multistage planning, for example: (i) Hierarchical Task Network (HTN) planning [21, 29, 30], (ii) network languages for complex systems [81]. In general, the complexity of the methods above is very high.

This article focuses on a special trajectory (multistage, discrete) decision making (DM) framework that is based on a digraph with five-types of vertices (nodes). The following issues are considered: (a) brief description of routing problem(s), (b) structuring the "design/solving space", and (c) heuristic solving schemes. A real world student trajectory problem (designing a student route from BS degree to PostDoc position) on the basis of a multicriteria orienteering model is examined (i.e., problem description and statement, solving heuristic, realistic numerical example). The paper is based on the preliminary electronic preprint [57] and the conference paper [58].

2. TRAJECTORY DECISION MAKING SCHEME

A generalized four-part scheme (morphological structure) of the examined domain (route/trajectory decision making problems) is depicted in Fig. 1.

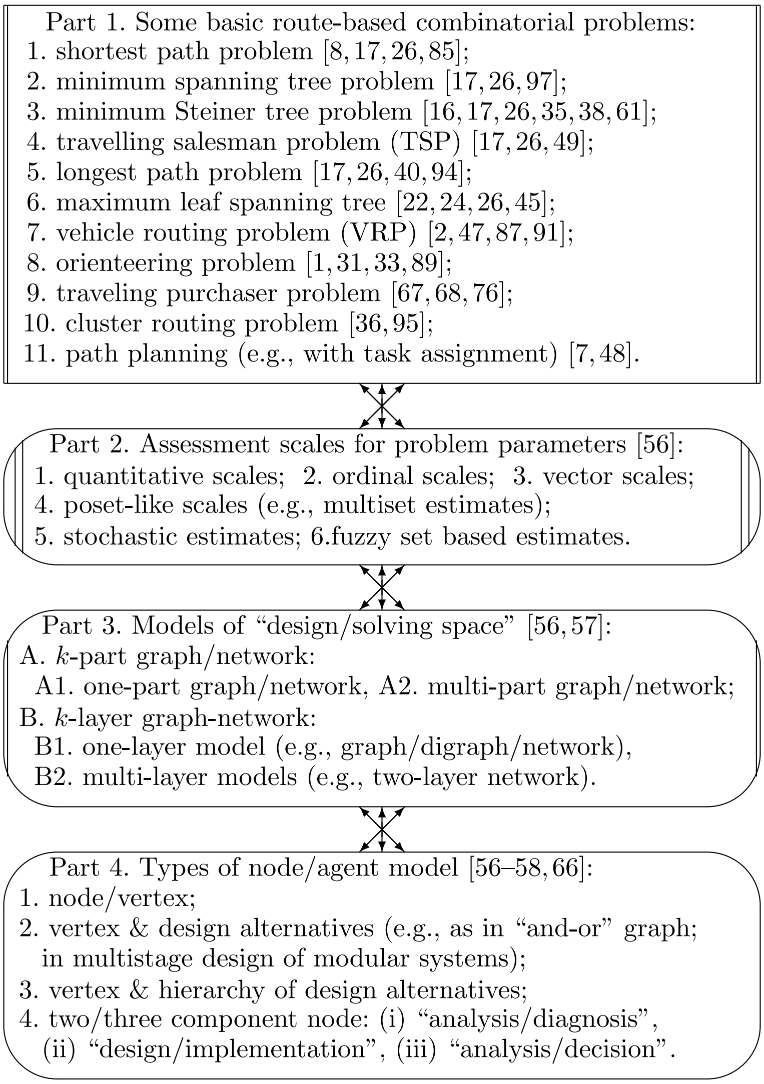


Fig. 1. Morphological scheme of trajectory/route DM problems

For example, two problem kinds can be pointed out as basic “physical” metaphors: (i) automobile routing problems (e.g., [10, 46]), (ii) team orienteering problems (e.g., [1, 31, 33, 42, 63, 89]). Table 1 contains a list of trajectory or route-like decision making problems. In the considered route DM problems, it is necessary to do the following: (1) generation of “design/solving space” (i.e., states, transmission operations); (2) specification of the goal as a resultant point (or a set of goal points); (3) design of the route at the “design/solving ‘space”, and (4) analysis of the route implementation and the route modification (if needed).

3. ROUTE DECISION MAKING PROBLEMS

3.1. Basic problems

An illustration for the basic routing decision making problem is depicted in Fig. 2. Here, directed graph $G = (H, E)$ is given (H is vertex set, E is arc set); initial vertex (origin) $h^0 \in H$ and goal vertex (destination) $h^g \in H$ are pointed out; each arc $e \in E$ has a length (i.e., nonnegative weight, cost) $\lambda(e)$. The basic problem is:

Find the route (path) from vertex h^0 to vertex h^g $L =< h^0, ..., h^g >$ that minimizes the total length (cost) of the path (i.e., the sum of the path arcs weights).

Table 1. Trajectory/route decision making problems

No.	Problem	Source(s)
1.	Basic "reference" applied problems:	
1.1.	"Physical" route (e.g., car/robot route)	[1, 46]
1.2.	Control trajectory in parameter space	[20, 71]
1.3.	Search strategy in problem solving	[64, 70]
1.4.	Urban route (choice, planning)	[5, 73, 77, 98]
1.5.	Trajectory for mobile robot movement	[6, 41]
1.6.	Mission of airplane/aerospace apparatus (e.g., unmanned aerial vehicles UAVs)	[39, 43, 44]
1.7.	Ship trajectories	[82, 83]
1.8.	Data path synthesis in digital systems (e.g., VLSI design)	[88]
1.9.	X-cast routing (anycast, broadcast, multi-broadcast, multicast, unicast, geocast)	[14, 23, 50, 56, 62]
2.	Prospective problems:	
2.1.	Motion planning, navigation (urban traffic planning, automobile routing, robot motion planning, inspection path planning, etc.)	[15, 41]
2.2.	Tourism route planning	[27, 28]
2.3.	Search trajectory in information systems	[9, 56]
2.4.	Scenario/multistage scenario	[11, 90]
2.5.	Trajectory of educational objects (e.g., student plan, course improvement trajectory, trajectory of research/educational center changes)	[52, 53]
2.6.	Plans in biomedicine (e.g., medical treatment, immunological analysis)	[53, 56, 58–60]
2.7.	Trajectory of modular system development	[53, 56]
2.8.	Trajectory of organizational-economical objects (e.g., start-up company, teams, objects (e.g., start-up company, team,	[56]
2.9.	Plans of monitoring processes (e.g., system testing/inspection, maintenance)	[43, 79]

Several polynomial algorithms were suggested for the problem (including polynomial algorithms for multi-objective versions) (e.g., [17, 26, 85]). Generally, the following support problems can be pointed out: (i) building the route (design), (ii) analysis of the route implementation and the online route modification (correction). Note, basic simplification approach consists in partitioning the initial solving "space" into series of "subspaces". Evidently, other versions of route decision making problems can be examined as well, for example: (a) multi-goal, multi-route problem (Fig. 3), (b) orienteering problem (Fig. 4), (c) route problem over multi-layer "solving space" (Fig. 5).

Some basic route DM models (and their variants) are well-known, for example (Fig. 1) [17,26,47, 87]: (a) traveling salesman problem(s); (b) longest path problem; (c) minimum spanning tree problem(s); (d) maximum leaf spanning tree problem; (e) vehicle routing problem(s) (VRPs). Further, the orienteering problem and its modifications will be used as a basic one (main applied domains: logistics, sport, tourism) (e.g., Fig. 3) [1, 28, 31, 89]. In fact, the problem integrates knapsack problem and TSP. Here, digraph $G = (H, E)$ ($|H| = n$) is given, each vertex $h \in H$ has a nonnegative score (profit) $\theta(h)$, each edge/arc $e \in E$ has a nonnegative length (cost, travel time) $\lambda(e)$.

The problem is:

Find a route (a path from the start point $h^0 \in H$ to the end point $h^g \in H$) over a subset of the most important graph vertices that maximizes the sum of the scores of the selected vertices while taking into account a constraint for total route length (total cost) (i.e., combination of knapsack problem and TSP).

The mathematical model is formulated as follows: $H = \{1, ..., i, ..., n\}$ is the set of vertex/nodes, vertex 1 is the start point of the route, vertex n is the end (goal) point of the route, binary variable $x_{ij} = 1$ if the built route (path) contains arc (i,j) and $x_{ij} = 0$ otherwise (vertex i precedes j), θ_i is the vertex profit, λ_{ij} is the arc cost (if arc $(i,j) \in E$), (d is a distance constraint for the built path). The model is:

$$\max \sum_{i=1}^{n} \sum_{j=1}^{n} \theta_i x_{ij}$$

$$s.t. \sum_{j=2}^{n} x_{1j} = \sum_{i=1}^{n-1} x_{in} = 1;$$

$$\sum_{i=2}^{n-1} x_{ik} = \sum_{j=2}^{n-1} x_{kj} \leq 1, \; k = \overline{2, n-1};$$

$$\sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_{ij} x_{ij} \leq d;$$

$$x_{ij} \in \{0,1\}, \; i = \overline{1,n}, \; j = \overline{1,n}.$$

The problem is NP-hard [31]. Multicriteria problem statement can be examined as well (e.g., the score of each vertex is a vector estimate and the objective function is a vector based on the score components summarization).

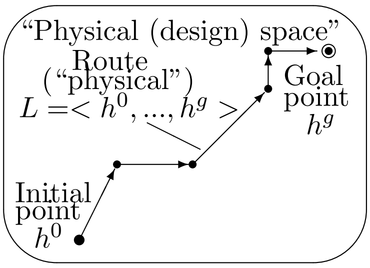


Fig. 2. Basic routing problem

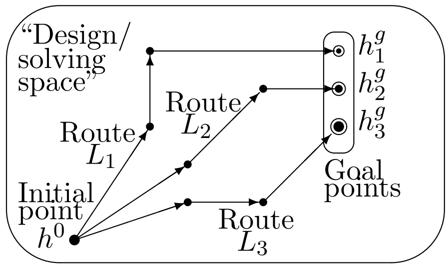


Fig. 3. Multi-goal, multi-route problem

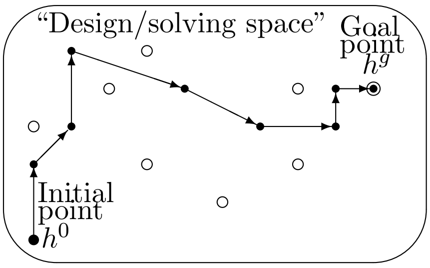


Fig. 4. Team orienteering problem

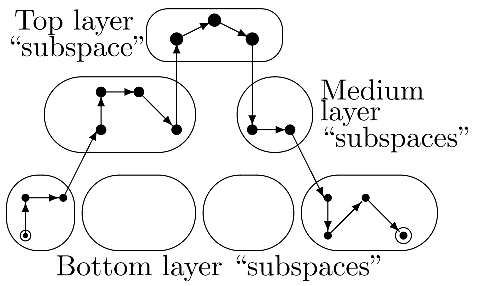


Fig. 5. 3-layer "design/solving space"

3.2. Structuring of design space

The "design/solving space" is modeled as a digraph/network. The following possible extensions of "design/solving space" are used [56, 57]: *1.* multi-layer structure of "design/solving space" (Fig. 5); *2.* multi-domain case (or multi-part digraph/'network) (Fig. 6); *3.* combined case. Three-domain trajectory/route DM problem based on different basic combinatorial optimization route problems (TSP, orienteering problem, shortest path) is depicted in Fig. 6.

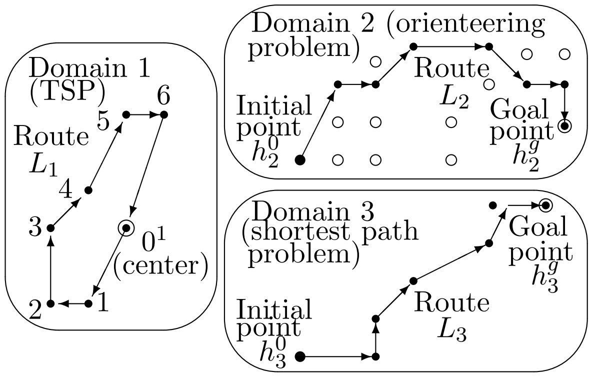


Fig. 6. Three-domain route DM problem

3.3. Types of nodes and illustration of routing

In general, the following five types of elements (i.e., node/vertex models) are considered (Fig. 7) [56–58, 66]:

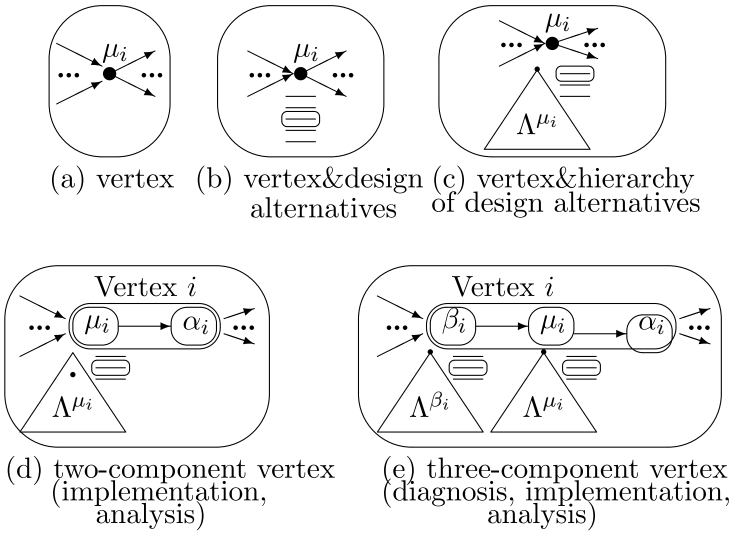


Fig. 7. Elements in "design/solving spaces"

1. Vertex/node (μ_i) (Fig. 7a). This case corresponds to traditional situation when a digraph is used (e.g., in the shortest path problem).

2. Vertex/node (μ_i) with corresponding design alternatives $\{A_1^{\mu_i}, ..., A_{q_{\mu_i}}^{\mu_i}\}$ (problem: selection of the best design alternative for the vertex) (Fig. 7b). This case can be used in routing in "and-or" digraphs [66], in network routing with selection of the best communication protocol at each network node [55, 56].

3. Vertex/node (μ_i) and corresponding hierarchy of design alternatives Λ^{μ_i} (problem: composition of the best composite design alternative(s) on the basis of hierarchy above) (Fig. 7c). This case can be used in network routing with hierarchical modular design of the implemented communication protocol at each node, in combinatorial planning of immunoassay technology [53, 55, 56, 60].

4. Composite (multi-component) vertex (i), for example: two components as follows: (a) "design/implementation" part (μ_i) (problem: composition of the best composite design alternative on the basis of hierarchy above to implement) and (b) "analysis/decision" part (α_i) (to analyze the result of the implementation above and selection of next way/path, usage of logical rules) (Fig. 7d).

This case can be used in combinatorial planning of medical treatment (i.e., design/implementation and analysis) [57, 58].

5. Composite (multi-component) vertex (i), for example: three components as follows: (a) "diagnosis" part (β_i) (problem: composition of the best composite diagnosis alternative on the basis of hierarchy, implementation of diagnosis procedure), (b) "design/implementation" part (μ_i) (problem: composition of the best composite design alternative on the basis of hierarchy above to implement), and (c) "analysis/decision" part (α_i) (to analyze the result of the implementation above and selection of next way/path, based on logical rules) (Fig. 7e). This case can be used in system maintenance, in medical treatment [58].

Two numerical examples illustrate the routing processes for case 2 and for case 3.

Example 1. An illustrative example for routing based on design alternatives at each graph/network vertex is the following. For each vertex, the resultant design alternative can be selected in online mode or on the basis of off-line solving process [56, 57]. Here, each vertex of "design space" corresponds to Fig. 7b. The example involves the following (Fig. 8):

(i) $G = (H, E)$ is a digraph, vertex set $H = \{\mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6, \mu_7, \mu_8\}$, arc set $E = \{(\mu_1, \mu_2)$, (μ_1, μ_3), (μ_2, μ_3), (μ_2, μ_4), (μ_2, μ_5), (μ_3, μ_5), (μ_3, μ_6), (μ_4, μ_6), (μ_3, μ_7), (μ_5, μ_6), (μ_5, μ_7), (μ_6, μ_7), (μ_6, μ_8), $(\mu_7, \mu_8)\}$;

(ii) μ_1 is an initial point, μ_8 is a goal point;

(iii) there exist three design alternatives for each vertex μ_i ($i = \overline{1,8}$): $A_1^{\mu_i}$, $A_2^{\mu_i}$,$A_3^{\mu_i}$;

(iv) selected alternatives (for each vertex) are: $A_3^{\mu_1}$, $A_3^{\mu_2}$,$A_1^{\mu_3}$, $A_2^{\mu_4}$, $A_2^{\mu_5}$,$A_1^{\mu_6}$, $A_1^{\mu_7}$, $A_2^{\mu_8}$ (in Fig. 9, the alternatives are pointed out by "oval");

(v) the designed global route (by vertices) is: $L =< \mu_1, \mu_2, \mu_5, \mu_6, \mu_8 >$; and

(vi) the resultant route consisting of design alternatives is: $\widehat{L} =< A_3^{\mu_1}, A_3^{\mu_2}, A_2^{\mu_5}, A_1^{\mu_6}, A_2^{\mu_8} >$.

In this problem, the selected design alternatives of neighbor path vertices have to be "good" compatible as in combinatorial synthesis approach (morphological clique problem) [53, 56].

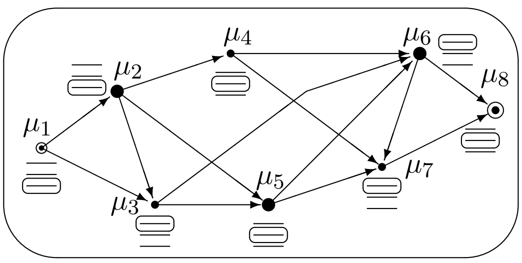


Fig. 8. Design alternatives for vertices

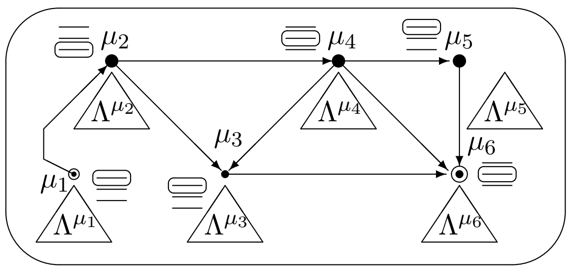


Fig. 9. Hierarchies of alternatives for vertices

Example 2. An illustrative example for routing based on hierarchy of design alternatives at each graph/network vertex is the following. For each vertex, the resultant set of design alternative can be designed in online mode or on the basis of offline solving process. Here, each vertex of "design space" corresponds to Fig. 7d. In [56], this problem is examined as multi-stage design of modular systems. The example involves the following (Fig. 9):

(i) $G = (H, E)$ is a digraph, vertex set $H = \{\mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6\}$, arc set $E = \{(\mu_1, \mu_2), (\mu_2, \mu_3)$, $(\mu_2, \mu_4), (\mu_3, \mu_6)$, (μ_4, μ_3), (μ_4, μ_5), (μ_4, μ_6), $(\mu_5, \mu_6)\})$;

(ii) μ_1 is an initial (origin) point, μ_6 is a goal (destination) point;

(iii) there exists a hierarchy of design alternatives for each vertex μ_i ($i = \overline{1,5}$): Λ^{μ_i};

(iv) three design alternatives are composed for each vertex μ_i ($i = \overline{1,6}$): $A_1^{\mu_i}$, $A_2^{\mu_i}$,$A_3^{\mu_i}$;

(v) selected alternatives (for each vertex) are: $A_1^{\mu_1}$, $A_3^{\mu_2}$,$A_1^{\mu_3}$, $A_2^{\mu_4}$, $A_1^{\mu_5}$, $A_2^{\mu_6}$ (in Fig. 9, the alternatives are pointed out by "oval");

(vi) the designed global route (by vertices) is: $L =< \mu_1, \mu_2, \mu_4, \mu_5, \mu_6 >$; and

(vii) the resultant route consisting of design alternatives is: $\widehat{L} =< A_1^{\mu_1}, A_3^{\mu_2}, A_2^{\mu_4}, A_1^{\mu_5}, A_2^{\mu_6} >$.

In this problem, the selected design alternatives of neighbor path vertices have to be "good" compatible (combinatorial synthesis based on morphological clique problem) [52, 53, 56].

4. BASIC SOLVING STRATEGIES

For basic routing problems, the following evident strategy can be used:

Strategy 0. Evident route strategy:

(0.1) design of a set of routes,

(0.2) selection of the best route.

In the case of complex nodes, the solving strategy has to contain special stages for analysis and selection/design of design alternatives at each node/vertex of the "solving space". Let us consider a one-layer route DM problem with nodes as "vertex& alternatives" and "vertex& hierarchy alternatives". The following two basic solving strategies can be pointed out for the problem type:

Strategy 1. "Global" route strategy:

(1.1) designing a "global" route over graph vertices,

(1.2) selection/design of the best design alternative for each graph vertex of the "global" route,

(1.3) composition of a resultant route from the best alternatives for each graph vertex of the "global" route.

Strategy 1 is illustrated in Example 1 (Fig. 8) [56, 57]. The approach was used for multi-stage design of modular systems [56].

Strategy 2. Extended digraph strategy:

(2.1) transformation/extension of the initial digraph (e.g., additional arcs, modification/extension of the alternatives hierarchies for the node(s)),

(2.2) designing the best route over the obtained extended graph/network.

Clearly, this strategy increases the problem dimension.

5. STUDENT EDUCATIONAL TRAJECTORY

In recent years, the attention was paid for educational decision making problems (e.g., curriculum design, students performance analysis, classification of students, composition of modular courses, program evaluation, student careers analytics, analysis of paths in student databases, student team formation, student careers planning, educational data mining, etc.) (e.g., [3, 4, 13, 18, 19, 34, 53, 54, 72, 74, 75, 78, 92, 93]). Here a simplified personalized plan (educational trajectory) for a BS student of Moscow Inst. of Physics and Technology (State Univ.) (Faculty of Radio Engineering and Cybernetics) is examined [57]:

Route from BS degree point to PostDoc position.

The morphological scheme of the problem is: (i) orienteering problem, (ii) ordinal and vector estimates, (iii) six-part digraph, (iv) simple node model. A BS degree in Communication Engineering is considered as initial point a_1 (origin), a PostDoc position in an university is considered as the goal point (destination). Table 2 contains educational points/nodes of the "solving space" including their characteristics and estimates upon criteria (ordinal scale $[1,5]$, 5 corresponds to the best level): (a) quality of educational program (i.e., a set of disciplines, basic lectures, seminars) C_1 (estimate θ^1), (b) possible research results (including publication activity) C_2 (estimate θ^2), and (c) integrated index of professional degree prestige (World University Rating, quality of professional education/research, scientific school(s), etc.) C_3 (estimate θ^3).

Table 2. Digraph model nodes/vertices for student trajectory

Node/ vertex	Degree level	Educational institution	Professional domain	Time (years) τ	Estimates θ^1	θ^2	θ^3
a_1	BS	MIPT, Russia	Commun. Eng.	4			
b_1	MS	MIPT, Russia	Commun. Eng.	2	5	2	4
b_2	MS	MIPT, Russia	Appl. Math.	2	5	4	4
b_3	MS	USA univ.	Commun. Eng.	2	3	5	5
b_4	MS	USA univ.	Inform. Syst.	2	3	5	5
b_5	MS	Canadian univ.	Commun. Eng.	2	3	4	4
b_6	MS	UK univ.	OR/Algorithms	2	4	5	4
b_7	MS	German univ.	Inform. Syst.	2	3	2	4
g_1	MS (2nd)	USA univ.	Commun. Eng.	1	3	5	5
g_2	MS (2nd)	USA univ.	Inform. Syst.	1	3	5	5
g_3	MS (2nd)	Canadian univ.	Commun. Eng.	1	3	4	4
g_4	MS (2nd)	UK univ.	OR/Algorithms	1	4	5	4
g_5	MS (2nd)	German univ.	Inform. Syst.	1	3	4	4
w_1	PhD	MIPT, Russia	Commun. Eng.	3	5	3	4
w_2	PhD	MIPT, Russia	Appl. Math.	3	5	4	5
w_3	PhD	USA univ.	Commun. Eng.	3	4	5	5
w_4	PhD	USA univ.	Inform. Syst.	3	4	5	5
w_5	PhD	Canadian univ.	Commun. Eng.	3	4	4	5
w_6	PhD	UK univ.	OR/Algorithms	3	5	5	5
w_7	PhD	German univ.	Inform. Syst.	3	4	4	4
f_1	PhD (2nd)	USA univ.	Commun. Eng.	2	4	5	5
f_2	PhD (2nd)	USA univ.	Inform. Syst.	2	4	5	5
f_3	PhD (2nd)	Canadian univ.	Commun. Eng.	2	4	5	5
p_1	PostDoc	USA univ.	Commun. Eng.	3	3	5	5

In Fig. 10, the examined simplified "design/solving space" (as series of subspaces; expert judgment) and five route examples are depicted:

$L^{ex_1} =< a_1, b_1, w_1, p_1 >$, $L^{ex_2} =< a_1, b_3, g_2, w_2, p_1 >$, $L^{ex_3} =< a_1, b_5, g_4, w_5, f_2, p_1 >$,

$L^{ex_4} =< a_1, b_6, g_5, w_6, p_1 >$, and $L^{ex_5} =< a_1, b_7, g_7, f_3, p_1 >$.

Further, four basic "generalized" educational trajectiories/routes are examined ($\iota = \overline{1,7}$, $\kappa = \overline{1,5}$, $\upsilon = \overline{1,7}$, $\xi = \overline{1,3}$) (expert judgment):

$L^1 =< a_1, b_\iota, w_\upsilon, p_1 >$ (Fig. 11a);

$L^2 =< a_1, b_\iota, g_\kappa, w_\upsilon, p_1 >$ (Fig. 11b);

$L^3 =< a_1, b_\iota, w_\upsilon, f_\xi, p_1 >$ (Fig. 11c); and

$L^4 =< a_1, b_\iota, g_\kappa, w_\upsilon, f_\xi, p_1 >$ (Fig. 11d).

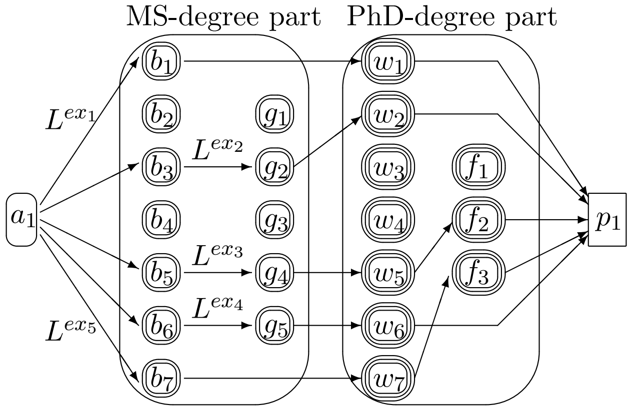
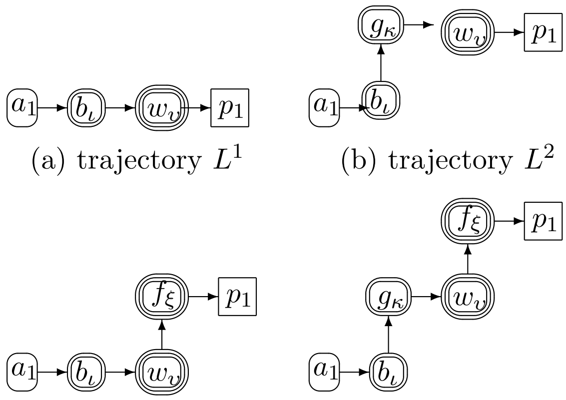


Fig. 10. "Design/solving space" (student trajectories)



(a) trajectory L^1 (b) trajectory L^2

(c) trajectory L^3 (d) trajectory L^4

Fig. 11. Basic general educational trajectories

Table 3, Table 4, Table 5 contain ordinal complexity estimates of movement between model nodes (scale $[1, 5]$, 5 corresponds to the most complex movement; absence of estimate corresponds to impossible movement: the digraph arc is absent).

First, a modification of orienteering problem (three objective functions, constraint for maximum arc length, constraint for aggregated (summarized) time of visited vertices) is considered as follows: $H = \{1, ..., i, ..., n\}$ is the set of vertex/nodes, vertex 1 is the start point of the route (origin), vertex n is the end (goal) point of the route (destination), binary variable $x_{ij} = 1$ if the built route (path) contains arc (i, j) and $x_{ij} = 0$ otherwise (vertex i precedes j), θ_i is the vertex profit, λ_{ij} is the arc cost (if arc $(i, j) \in E$), d^{max} is a distance constraint for movement between neighbor vertices in the built route/path, T is a time constraint for the built route as summarization of time costs of path vertices.

The basic model is:

$$\max \sum_{i=1}^{n} \sum_{j=1}^{n} \theta_i^1 x_{ij}, \quad \max \sum_{i=1}^{n} \sum_{j=1}^{n} \theta_i^2 x_{ij}, \quad \max \sum_{i=1}^{n} \sum_{j=1}^{n} \theta_i^3 x_{ij},$$

$$s.t. \quad \sum_{j=2}^{n} x_{1j} = \sum_{i=1}^{n-1} x_{in} = 1; \quad \sum_{i=2}^{n-1} x_{ik} = \sum_{j=2}^{n-1} x_{kj} \leq 1, \; k = \overline{2, n-1};$$

$$\lambda_{ij} x_{ij} \leq d^{max} \quad \forall i, j; \quad \sum_{i=1}^{n} \sum_{j=1}^{n} \tau_i x_{ij} \leq T; \quad x_{ij} \in \{0, 1\}, \; i = \overline{1, n}, \; j = \overline{1, n}.$$

Table 3. Ordinal estimates of movement complexity $\lambda(a_1 \rightarrow b_\iota)$

$a_1 \setminus b_i$	b_1	b_2	b_3	b_4	b_5	b_6	b_7
a_1	1	2	4	5	4	5	3

Table 4. Ordinal estimates of movement complexity: $\lambda(b_\iota \rightarrow g_\kappa)$, $\lambda(b_\iota \rightarrow w_\upsilon)$

$b_\iota \setminus g_\kappa$, $b_\iota \setminus w_\upsilon$	g_1	g_2	g_3	g_4	g_5	w_1	w_2	w_3	w_4	w_5	w_6	w_7
b_1	1	2	1	3	2	1	3	3	4	3	5	4
b_2	1	1	1	1	1	2	1	4	4	4	4	3
b_3		1		1				1	2	1	3	1
b_4	1		1					2	1	1	3	1
b_5	1	1		1				2	2	1	3	1
b_6	1	1	1					1	1	1	1	1
b_7	1	1	1	1				2	2	2	3	1

Table 5. Ordinal estimates of movement: $\lambda(g_\kappa \rightarrow w_\upsilon)$, $\lambda(w_\upsilon \rightarrow f_\xi)$, $\lambda(w_\upsilon \rightarrow p_1)$, $\lambda(f_\xi \rightarrow p_1)$

$g_\kappa \setminus w_\upsilon$, $w_\upsilon \setminus f_\xi/p_1$, $f_\xi \setminus p_1$	w_1	w_2	w_3	w_4	w_5	w_6	w_7	f_1	f_2	f_3	p_1
g_1			1	2	1	3	1				
g_2			2	1	1	3	1				
g_3			3	3	1	3	1				
g_4			1	1	1	1	1				
g_5			2	2	2	3	1				
w_1								2	3	2	5
w_2								1	1	1	5
w_3									1		2
w_4								1		1	3
w_5									1		3
w_6								1	1	1	3
w_7								3	0	2	4
f_1											1
f_2											2
f_3											2

Here, the Pareto-efficient solutions have to be searched for. Clearly, the problem is NP-hard. In our case, a_1 is the start point (i.e., graph vertex), p_1 is the end (goal) point (i.e., graph vertex). The optimization model has to be solved for each generalized trajectory above (i.e., L^1, L^2, L^3, L^4).

Let $L =< l_1, ..., l_i, ..., l_q >$ be an admissible route solution (i.e., educational trajectory). The characteristics of the solution are as follows:

(a) $\theta^1(L) = \sum_{i=2}^{q} \theta_i^1$ is integrated quality of education;

(b) $\theta^2(L) = \sum_{i=2}^{q} \theta_i^2$ is integrated quality of research results (including publication results);

(c) $\theta^3(L) = \sum_{i=2}^{q} \theta_i^3$ is integrated parameter of resultant prestige of the obtained academic degrees;

(d) $\tau(L) = \sum_{i=2}^{q} \tau_i$ is integrated required time (years) of the educational trajectory;

(e) $d(L) = \sum_{i=1}^{q-1} \lambda(l_i \rightarrow l_{i+1})$ is integrated estimate of movement complexity (between neighbor educational institutions).

As a result, the problem can be formulated as the following:

Find the route L (solution) such that

(1) it fulfils two constraints: $\tau(L) \leq T$ and $d(L) \leq d^{max}$;

(2) it is a Pareto-efficient one by four criteria (objective functions):
$\max\ \theta^1(L),\ \max\ \theta^2(L),\ \max\ \theta^3(L),\ \min\ d(L)$.

Note, the usage of educational generalized trajectories (L^1, L^2, L^3, L^4) leads to simplified/partitioned "solving space(s)". As a result, the optimization problem can be transformed into a version of the multicriteria shortest path problem or multicriteria multiple choice problem. Thus, a "concurrent" general solving framework is used (Fig. 12).

In the example, a simplified heuristic is considered:

Stage 1. Searching for a solution with minimum $d(L)$ for each generalized trajectory (L^1, L^2,L^3, L^4). The resultant solutions and their estimates are presented in Table 6.

Stage 2. Selection of Pareto-efficient solutions:
$L^1_1 =< a_1, b_3, w_3, p_1 >$, $L^2_1 =< a_1, b_1, g_1, w_3, p_1 >$,
$L^3_1 =< a_1, b_2, w_2, f_1, p_1 >$, and $L^4_3 =< a_1, b_2, g_2, w_4, f_1, p_1 >$.

Stage 3. Selection of the best solution (i.e., expert judgment).

In the example, solution $L^3_1 =< a_1, b_2, w_2, f_1, p_1 >$ can be selected while taking into account obtained additional skills in applied mathematics (it may be crucial for the student future).

Table 6. Pareto-efficient solutions and their parameters (vector estimate)

Route L	$\theta^1(L)$	$\theta^2(L)$	$\theta^3(L)$	$\tau(L)$	$d(L)$
$L^1_1 =< a_1, b_3, w_3, p_1 >$	10	15	15	8	7
$L^2_1 =< a_1, b_1, g_1, w_3, p_1 >$	15	17	19	9	5
$L^3_1 =< a_1, b_2, w_2, f_1, p_1 >$	15	18	19	10	5
$L^4_1 =< a_1, b_1, g_1, w_7, f_1, p_1 >$	19	21	23	11	6
$L^4_2 =< a_1, b_1, g_3, w_3, f_2, p_1 >$	19	21	23	11	6
$L^4_3 =< a_1, b_2, g_2, w_4, f_1, p_1 >$	19	22	24	11	6

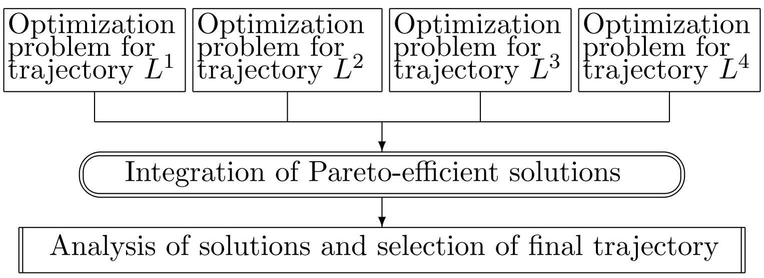


Fig. 12. Solving framework for educational trajectory

6. CONCLUSION

The paper suggests an approach (framework) for a special kind of multistage decision making based on a network with multi-type nodes. The approach is examined as a digraph based "intelligent" routing at special "design/solving space(s)". A realistic example for designing a student trajectory illustrates the framework.

Some future research directions are: *1.* analysis, modeling and usage of various kinds of "design/solving spaces" (including series composition of subspaces, dynamical "design/solving space"); *2.* usage of various basic combinatorial routing problems (e.g., spanning trees problems, versions of TSP, generalized orienteering problem) for construction of the corresponding trajectory/route DM problems; *3.* study of multi-layer (hierarchical) "design (solving) spaces" and trajectory (route) DM problems over them; *4.* special investigation of multiple vehicle routing problems (i.e., multi-domain problems) including coordination solving modes (e.g., as in multi-robot motion planning problems, in cooperative path planning for multiple UAVs); *5.* usage of multistage DM problems for testing (inspection, maintenance) of networked systems; *6.* applications of the examined multistage DM problems in economics/ management, e.g., modeling of firm (or project) development, forecasting, scenario planning; *7.* design of a special support computer-aided system for the considered multistage DM problems; and *8.* usage of the considered multistage DM problems in education (CS, applied mathematics, engineering, management).

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REFERENCES

1. Archetti C, Hertz A, Speranza MG, Metaheuristics for the team orienteering problem. *J of Heuristics*, 2017, vol. 13, pp. 49–76.
2. Archetti C, Speranza MG, The split delivery vehicle routing problem: a survey. In: Golden BL, Raghavan R, Wasil E (eds), *The Vehicle Routing Problem: Latest Advances and New Challenges.* Berlin: Springer, pp. 103–122, 2018.
3. Bacon DR, Stewart KA, Silver WS, Lessons from the best and worst student team experiences: how a teacher can make the difference. *J. of Manag Educ*, 1999, vol. 23, no. 5, pp. 467–488.
4. Baker RSJD, Educational data mining: an advance for intelligent systems in education. *IEEE Intell Syst*, 2014, vol. 29, no. 3, pp. 78–82.
5. Bekhor S, Ben-Akiva M, Ramming MS, Evaluation of choice set generation algorithms for route choice models. *Ann Oper Res*, 2006, vol. 144, no. 1, pp. 235–247.
6. Biagiotti L, Melchiorri C, *Trajectory Planning for Automatic Machines and Robots.* Berlin: Springer, 2008.
7. Biswas S, Anavatti SG, Garratt MA, A time-efficient co-operative path planning model combined with task assignment for multi-agent systems. *Robotics*, 2019, vol. 8, art. 35.
8. Boffey B, Fernandez Garcia FR, Laporte G, Mesa JA, Pelegrin BP, Multiobjective routing problems. *Top*, 1995, vol. 3, no. 2, pp. 167–220.
9. Bose P, Czyzowicz J, Gasieniec L, Kranakis E, Krizanc D, Pelc A, Martin MV, Strategies for hotlink assignments. In: *Int. Symp. on Algorithms and Computation (ISAAC'00)*, LNCS 1969, Berlin: Springer, pp. 23–34, 2001.
10. Boyce DE, Route guidance systems for improving urban travel and location choices. *Transp. Res. - A*, 1988, vol. 22, no. 4, pp. 275–281.
11. Bradfield R, Wright G, Burt G, Cairns G, van der Heijden K, The origins and evolution of scenario techniques in long range business planning. *Futures*, 2005, vol. 37, no. 8, pp. 795–812.
12. Buede DM, Miller WD, *The Engineering Design of Systems: Models and Methods.* 3rd ed., New York: Wiley, 2016.
13. Campagni R, Merlini D, Sprugnoli R, Verri MC, Data mining models for student careers. *Exp Syst with Appl*, 2015, vol. 42, no. 13, pp. 5508–5521.
14. Cavendish D, Gerla M. Routing optimization in communication networks. In: Cheng MX, Li Y, Du D-Z (eds), *Combinatorial Optimization in Communication Networks.* Berlin: Springer, pp. 505–547, 2006.
15. Choset HM, Lynch KM, Hutchinson S, Kantor G, Burgard W, Kavraki LE, Thrun S, *Principles of Robot Motion: Theory, Algorithms, and Implementations.* Boston: MIT Press, 2005.
16. Cieslik D, *Steiner Minimal Trees.* New York: Springer, 1998.

17. Cormen TH, Leiserson CE, Rivest RL, Stein C, *Introduction to Algorithms.* 3rd ed., Boston: MIT Press and McGraw-Hill, 2009.

18. Crawley EF, Malmqvist J, Ostlund S, Brodeur DR, Edstrom K, *Rethinking Engineering Education.* 2nd ed., Berlin: Springer, 2014.

19. Deibel K, Team formation methods for increasing interaction during in-class group work. *ACM SIGCSE Bulletin*, 2005, vol. 37, no. 3, pp. 291–295.

20. Dorf RC, Bishop RH, *Modern Control Systems.* 12th ed., Boston: Addison-Wesley, 1998.

21. Erol K, Hendler J, Nau DS, Complexity results for HTN planning. *Ann Math Artif Intell*, 1996, vol. 18, pp. 69–93.

22. Fernau H, Kneis J, Krasch D, Langer A, Liedloff M, Raible D, Rossmanith P, An exact algorithm for the Maximum Leaf Spanning Tree problem. *Theor Comp Sci*, 2011, vol. 412, no. 45, pp. 6290–6301.

23. Forouzan BA, *Data Communications and Networking.* 5th ed., New York: McGraw-Hill, 2012.

24. Fujie T, An exact algorithm for the maximum leaf spanning tree problem. *Comp Oper Res*, 2003, vol. 30, no. 13, pp. 1931–1944.

25. Garrido A, Barber F, Integrating planning and scheduling. *Int J Appl Artif Intell*, 2011, vol. 15, no. 5, pp. 471–491.

26. Garey MR, Johnson DS. *Computers and Intractability.* San-Francisco: W.H. Freeman, 1979.

27. Gavalas D, Konstantopoulos C, Mastakas K, Pantziou G, Vathis N, Heuristics for the time dependent team orienteering problem: application to tourist route planning. *Comp Oper Res*, 2015, vol. 62, pp. 36–50.

28. Gavalas D, Kasapakis V, Konstantopoulos C, Pantziou G, Vathis N, Zaroliagis C, The eCOMPASS multimodal tourist tour planner. *Exp Syst with Appl*, 2015, vol. 42, no. 21, pp. 7303–7316.

29. Georgievsk I, Aiello M, An overview of hierarchical task network planning. Elec. prepr. 45 p., Mar. 28, 2014. http://arxiv.org/abs/1403.7426 [cs.AI]

30. Ghallab M, Nau D, Traverso P, *Automated Planning: Theory and Practice.* Burlington: Morgan Kaufmann, 2004.

31. Golden BL, Levy L, The orienteering problem. *Nav Res Log*, 1987, vol. 34, pp. 307–318.

32. Gonzalez C, Decision support for real-time dynamic decision making tasks. *Org Beh and Human Dec Proc*, 2005, vol. 96, no. 2, pp. 142–154.

33. Gunawan A, Lau HC, Vansteenwegen P, Orienteering problem: a survey of recent variants, solution approaches and applications. *Eur J of Oper Res*, 2016, vol. 255, no. 2, pp. 699–718.

34. Guruler H, Istanbullu A, Karahasan M, A new student performance analizing system using knowledge discovery in higher educational databases. *Comp Educ*, 2010, vol. 5, no. 1, pp. 247–254.

35. Gutin G, Punnen AP (eds), *The Traveling Salesman Problem and Its Variations.* Berlin: Springer, 2007.

36. Guttmann-Beck N, Hassin R, Khuller S, Raghavachari R, Approximation algorithms with bounded performance guarantees for the clustered traveling salesman problem. *Algorithmica*, 2000, vol. 28, no. 4, pp. 422–437.

37. Hotaling JM, Fakhari P, Busemeyer JR, Dynamic decision making. In: Wright JD (ed), *Int. Encyclopedia of the Social & Behavioral Sciences.* 2nd ed., vol. 6, Amsterdam: Elsevier, pp. 709–714, 2015.

38. Hwang FK, *The Steiner Tree Problem.* Amsterdam: North Holland, 2012.

39. Jun M, D'Andrea R, Path planning for unmanned aerial vehicles in uncertain and adversarial environments. In: Butenko S, Murphey R, Pardalos P (eds), *Cooperative Control Models, Applications and Algorithms.* Berlin: Springer, pp. 95–110, 2003.

40. Karger D, Motwani R, Ramkumar GDS, On approximating the longest path in a graph. *Algorithmica*, 1997, vol. 18, no. 1, pp. 82–98.

41. Kavraki LE, Svestka P, Latombe J-C, Overmars MH, Probabilistic roadmaps for path planning in high-dimensional configuration spaces. *IEEE Trans Rob Autom*, 1996, vol. 14, no. 4, pp. 566–580.

42. Ke L, Zhai L, Li J, Chan FTS, Pareto mimic algorithms: an approach to the team orienteering problem. *Omega*, 2016, vol. 61, pp. 155–166.

43. Kim SC, Shepperd SW, Norris HL, Goldberg HR, Wallace MS, Mission design and trajectory analysis for inspection of a host spacecraft by a microsatellite. In: *2007 IEEE Aerospace Conf*, pp. 1–23, 2007.

44. Kim H, Park B, Nyung H, Curvature path planning with high resolution graph for unmanned surface vehicle. In: *Advances in Intell Syst and Comput*, vol. 208. Berlin: Springer, pp. 147–154, 2013.

45. Kleitman DJ, West DB, Spanning trees with many leaves. *SIAM J Discr Math*, 1991, vol. 4, pp. 99–106.

46. Kumar P, Singh V, Reddy D, Advanced traveler information system for Hyderabad City. *IEEE Trans Intell Transp Syst*, 2005, vol. 6, no. 1, pp. 26–37.

47. Laporte G, Fifty years of vehicle routing. *Transp Sci*, 2009, vol. 43, no. 4, pp. 408–416.

48. Lavalle S, *Planning Algorithms*. New York: Cambridge Univ. Press, 2006.

49. Lawer EL, Lenstra JK, Rinnooy Kan AHG, Shmoys DB, *The Traveling Salesman Problem: A Guided Tour of Combinatorial Optimization*. New York: Wiley, 1985.

50. Lee S-H, Ko Y-B, Efficient geocasting with multi-target regions in mobile multi-hop wireless networks. *Wireless Netw*, 2010, vol. 16, no. 5, pp. 1253–1262.

51. Levin L, Kowalski DR, Segal M, Message and time efficient multi-broadcast schemes. *Theor Comp Sci*, 2015, vol. 569, pp. 13–23.

52. Levin MS, *Combinatorial Engineering of Decomposable Systems*. New York: Springer, 1998.

53. Levin MS, *Composite Systems Decisions*. London: Springer, 2006.

54. Levin MS, Towards combinatorial evolution of composite systems. *Exp Syst with Appl*, 2013, vol. 40, no. 4, pp. 1342–1351.

55. Levin MS, A modular approach to the communication protocol and standard for multimedia information: a review. *J Commun Techn Electr*, 2013, vol. 58, no. 6, pp. 594–601.

56. Levin MS, *Modular System Design and Evaluation*. Cham: Springer, 2015.

57. Levin MS, Discrete route/trajectory decision making problems. Elec. prepr. 25 p., Aug. 16, 2015. http://arxiv.org/abs/1508.03863 [cs.AI]

58. Levin MS, Digraph based medical treatment planning. In: *Proc. of 2015 Int Conf on Biomedical Engineering and Computational Technologies*, pp. 171–175, 2015.

59. Levin MS, Sokolova LV, Hierarchical combinatorial planning of medical treatment. *Comp Progr Meth Biomed*, 2004, vol. 74, no. 1, pp. 3–11.

60. Levin MS, Firer M, Hierarchical morphological design of immunoassay technology. *Comp Biol Med*, 2005, vol. 35, no. 3, pp. 229–245.

61. Levin MS, Zamkovoy AA, Multicriteria Steiner tree with the cost of Steiner vertices. *J of Commun Techn Electr*, 2011, vol. 56, no. 12, pp. 1527–1542.

62. Lilienthal S, Mandjes M, Flow-level models for multipath routing. *Perfor Eval*, 2011, vol. 68, no. 7, pp. 551–574.

63. Lin S-W, Yu VF, Solving the team orienteering problem with time windows and mandatory visits by multi-start simulated annealing. *Comp Ind Eng*, 2017, vol. 114, pp. 196–205.

64. Luger GF, *Artificial Intelligence: Structures and Strategies for Complex Problem Solving*. 5th ed., Boston: Addison-Wesley, 2004.

65. Lupeikiene A, Dzemyda G, Kiss F, Carplinskas A, Advanced planning and scheduling systems: Modeling and implementation challenges. *Informatica*, 2014, vol. 25, no.4, pp. 581–616.

66. Mahanti A, Bagchi A, AND/OR graph heuristic search methods. *J of the ACM*, 1985, vol. 32, no. 1, pp. 28–51.

67. Manerba D, Mansini R, A branch-and-cut algorithm for the Multi-vehicle Traveling Purchaser Problem with Pairwise Incompatibility Constraints. *Networks*, 2015, vol. 65, no. 2, pp. 139–154.

68. Manerba D, Mansini R, Riera-Ledesma J, The Traveling Purchaser Problem and its variants. *Eur J Oper Res*, 2017, vol. 259, no. 1, pp. 1–18.

69. Murata T, Petri nets: properties, analysis and applications. *Proc of the IEEE*, 1989, vol. 77, no. 4, no. 541–558.

70. Nilsson NJ, *Problem Solving Methods in Artificial Intelligence*. New York: MacGraw Hill, 1971.

71. Nise NS, *Control Systems Engineering*. 7th. ed., New York: Wiley, 2015.

72. Oakley B, Felder RM, Brent R, Elhajj I, Turning student groups into effective teams. *J of Student Centered Learn*, 2004, vol. 2, no. 1, pp. 9–34.

73. Papinski D, Scott DM, A GIS-based toolkit for route choice analysis. *J Trans Geogr*, 2011, vol. 19, pp. 434–442.

74. Pena-Ayala A, Educational data mining: a survey and a data mining-based analysis. *Exp Syst with Appl*, 2014, vol. 41, no. 4, Part 1, pp. 1432–1462.

75. Rad A, Naderi B, Soltani M, Clustering and ranking university majors using data mining and AHP algorithms. A case study in Iran. *Exp Syst with Appl*, 2011, vol. 38, no. 1, pp. 755–763.

76. Rierra-Ledesma J, Salazar-Gonzalez J-J, Solving the asymmetric traveling purchaser problem. *Ann Oper Res*, 2006, vol. 144, no. 1, pp. 83–97.

77. Ringhand M, Vollrath M, Make this detour and be unselfish! Influencing urban route choice by explaining traffic management. *Trans Res Part F*, 2018, vol. 53, pp. 99–116.

78. Romero C, Ventura S, Educational data mining: a survey from 1995 to 2005. *Exp Syst with Appl*, 2007, vol. 33, no. 1, pp. 135–146.

79. Shchurov AA, Marik R, Khlevnoy VA, A formal approach to network/distrubuted systems complex testing. *Int J of Comp Trends Technol*, 2015, vol. 22, no. 2, pp. 76–80.

80. Stadlter H, Kilger C, Meyer H (eds), *Supply Chain Management and Advanced Planning. Concepts, Models, Software and Case Studies*. 5th ed., Berlin: Springer, 2008.

81. Stilman B, Network languages for complex systems. *Int J Comp Math Appl*, 1993, vol. 26, no. 8, pp. 51–79.

82. Szlapczynski R, A new method of ship routing on raster grids, with turn penalties and collision avoidance. *J Navig*, 2006, vol. 59, pp. 27–42.

83. Szlapczynski R, Szlapczynska J, On evolutionary computing in multi-ship trajectory planning. *Appl Intell*, 2012, vol. 37, pp. 155–174.

84. Tamosaitiene J, Zavadskas EK, The multi-stage decision making systems for complicated problems. *Procedia - Soc Behav Sci*, 2013, vol. 82, pp. 215–219.

85. Tarapata Z, On a multicriteria shortest path problem. *Int J Appl Math Comput Sci*, 2007, vol. 17, no. 2, pp. 269–287.

86. Tarsensko S, Modeling multistage decision processes with reflexive game theory. Elec. prepr. 8 p., Mar. 11, 2012. http://arxiv.org/abs/1203.2315 [cs.MA]

87. Toth P, Vigo D, *The Vehicle Routing Problem. Monographs on Discrete Mathematics and Applications*. Philadelphia: SIAM, 2001.

88. Tseng C-J, Siewiorek DP, Automated synthesis of data path in digital synthesis. *IEEE Trans Comp-Aided Des*, 1986, vol. 5, no. 3, pp. 379–395.

89. Vansteenwegen P, Souffriau W, Van Oudheusden D, The orienteering problem: A survey. *Eur J Oper Res*, 2011, vol. 209, no. 1, pp. 1–10.

90. Varum CA, Melo C, Directions in scenario planning literature - A review of the past decades. *Futures*, 2010, vol. 42, no. 4, pp. 355–369.

91. Vidal T, Crainic TG, Gendreau M, Prins C, A unified solution framework for multi-attribute vehicle routing problems. *Eur J Oper Res*, 2014, vol. 234, no. 3, pp. 658–673.

92. Xing W, Guo R, Petakovic E, Goggins S, Participation-based student final performance prediction model through interpretable Genetic Programming: Integrating learning analytics, educational data mining and theory. *Comp Human Behav*, 2015, vol. 47, pp. 168–181.

93. Zafra A, Romero C, Ventura S, Multiple instance learning for classifying students in learning management systems. *Exp Syst with Appl*, 2011, vol. 38, no. 12, pp. 15020–15031.

94. Zhang Z, Li H, Algorithms for long paths in graphs. *Theor Comp Sci*, 2007, vol. 377, pp. 25–34.

95. Zhang X, Du D, Gutin G, Ming Q, Sun J, Approximating algorithms for general cluster routing problem. In: *CoCOON 2020*, Berlin: Springer, pp. 472–483, 2020.

96. Zhou M, Venkatesh K, *Modelling, Simulation, & Control of Flexible Manufacturing Systems: A Petri Net Approach*. Singapore: World Scientific Pub., 1998.

97. Zhou G, Gen M, Genetic algorithm approach on the multi-criteria minimum spanning tree problem. *Eur J Oper Res*, 1999, vol. 114, no. 1, pp. 141–152.

98. Zhu C, Hu JQ, Wang F, Xu Y, Cao R, On the tour planning problem. *Ann Oper Res*, 2012, vol. 192, no. 1, pp. 67–86.