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О задаче p-центров

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Аннотация—В статье рассмотрена задача p-центров (и задача p-медиан) для графов/сетей. В целом, материал основан на архитектурном взгляде на область задач p-центров и включает следующие части: (1) обзор по задачам p-центров и подходам к их решению, некоторые комбинаторные постановки этих задач включая многокритериальную версию; (2) обзор по задачам p-медиан и подходам к их решению, некоторые комбинаторные постановки этих задач включая многокритериальные версии; (3) иллюстративные численные примеры; (4) новая прикладная задача выделения "центральных"статей в группе статей по тематике систем связи.

КЛЮЧЕВЫЕ СЛОВА: задача р-центров, задача р-медиан, комбинаторная оптимизация, эвристики, приложение

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1. ВВЕДЕНИЕ

Статья посвящена следующим вопросам: (1) представлены литературные обзоры по задачам p-центров (или k-центров) и по задачам p-медиан, приведены классификации задач и методов их решения; (2) приведены модели комбинатоной оптимизации (включая многокритериальные постановкик); (3) кратко описаны упрощенные эвристические схемы; (4) представлены иллюстративные цифровые примеры; (5) предложена новая прикладная задача выделения "центральных"статей в группе статей по тематике систем связи. Задачи p-центров и p-медиан могут рассматриваться как часть области комбинаторной кластеризации [1].

2. ЗАДАЧИ Р-ЦЕНТРОВ

2.1. Описание и иллюстрации

Классическая задача p-центров (или k-центров) заключается в выборе подмножества из p вершин (центры, пункты обслуживания, пункты размещения оборудования) в ненаправленном графе с целью минимизировать максимальное расстояние между каждой вершиной графа (пользователь, клиент) (из множества не-центров данного графа) и ближайшим центром [2, 3, 4, 5, 6]. Данная задача близка к задаче покрытия всех вершин графа не более p кругами наименьшего радиуса. Задача p-центров относится к классу NP-трудных задач [2, 4, 5, 7]. Данная задача широко используется в различных областях (таблица 1).

В литературе рассматриваются следующие основные версии задачи:

1. Дано множество из n точек в некотором метрическом пространстве [4, 8, 9]:

Найти p специальных точек (центров) так, чтобы максимальная дистанция от остальных точек до центров была минимальной.

2. В непрерывной задаче p-центров кластеризации центры могут быть расположены в любом месте исходного пространства [8, 10, 11, 12].

3. В специальной дискретной версии задачи p-центров множество центров-кандидатов задается заранее [8].

4. В задаче с минимальным покрытием исходное пространство покрывается p шарами так, чтобы радиус наибольшего шара был минимален [13].

Таблица 1. Некоторые приложения задач p-центров

Ном.	Задача	Источники
1.	Размещение сервисных центров:	
1.1.	Размещение логистических центров (в частности, на древовидных структурах)	[14, 15]
1.2.	Размещение медицинских центров	[16]
1.3.	Размещение центров (для хранения базового контента) в Интенете	[17]
2.	Размещение оперативных центров (экстренной помощи, и т.п.):	
2.1.	Размещение центров неотложной медицинской помощи	[18]
2.2.	Определение расположения госпиталей	[19]
2.3.	Размещение пожарных депо	[20, 21]
3.	Приложения в сетях (транспорт, связь):	
3.1.	Размещение центров (хабов) в сетях	[22]
3.2.	Размещение центров распределения, коммутации в сетях связи	[5, 19, 23, 24]
3.3.	Размещение сенсоров в сетях	[25]
3.4.	Размещение оборудования в транспортных (городских)) сетях	[26, 23, 27]
3.5.	Размещение оборудования в мобильных сетях связи	[28]
4.	Некоторые другие приложения:	
4.1.	Размещение центров интеграции данных	[29]
4.2.	Анализ логистичесих сетей (поставщики-потребители)	[30]
4.3.	Определение "центральности"в сетях цитирования научных публикаций	[31]
4.4.	Анализ в реляционных базах данных	[32]

Базовая задача k-центров (т.е., известная задача размещения оборудования в k-центрах) описана в литературе [2, 3, 13, 33, 34, 35]. Имеется полный ненаправленный граф $G = (A, E)$, A - множество вершин, E - множество ребер. Рассматривается метрика d: $A \times A \to R^+$. Задача имеет вид:

Найти подмножество $B \subseteq A$ из максимум k центров (k - положительное целое) которое минимизирует максимальное расстояние от элементов из A к B. Используется целевая функция:

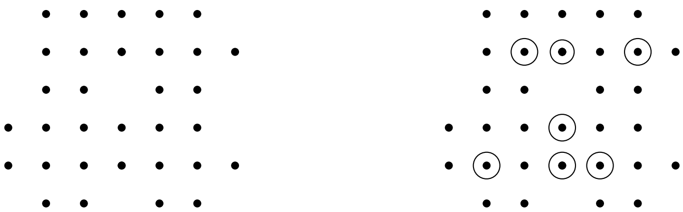
$$\min_{B \subseteq A, |B| \leq k} \max_{a \in A} \min_{b \in B} d(a, b)$$

Эта модель комбинаторной оптимизации является NP-трудной [4, 5]). Вместо расстояния d могут рассматриваться различные типы близости (например, порядковые или векторные близости, вероятностные и размытые оценки).

Упрощенные иллюстрации для задачи приведены на Рис. 1 и Рис. 2:

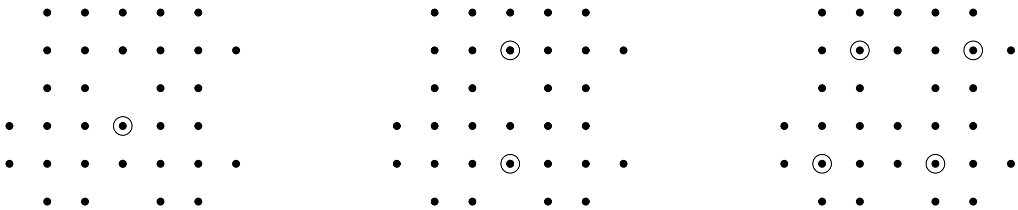
(i) исходные данные: (a) множество элементов (вершин) (Рис. 1a), (b) центры-кандидаты (Рис. 1b);

(ii) иллюстративные примеры решений: (a) решение: 1-центр (Рис. 2a), (b) решение: 2-центра (Рис. 2b), (c) решение: 4-центра (Рис. 2c).



(a) исходные элементы (b) центры-кандидаты

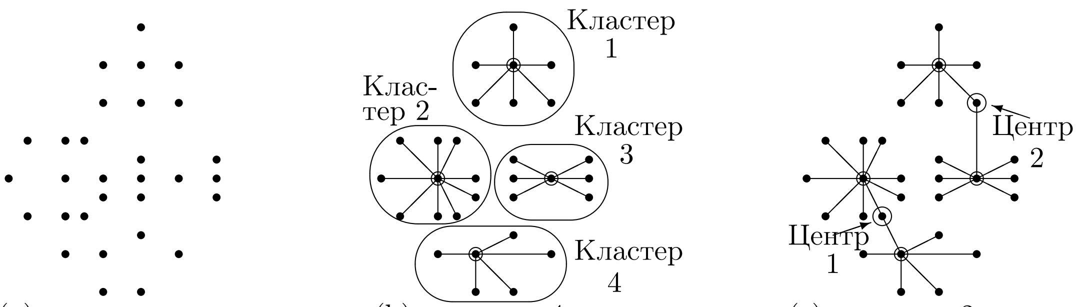
Рис. 1. Исходные данные для примера задачи p-центров



(a) решение: 1-центр (b) решение: 2-центра (c) решение: 4-центра

Рис. 2. Иллюстрация решения для задачи p-центров

Отметим, что процесс решения задачи p-центров может быть основан на кластеризации (т.е., предварительном формировании p кластеров) (Рис. 3).



(a) исходные элементы (b) решение: 4-центра (c) решение: 2-центра

Рис. 3. Задача p-центров на основе кластеризации

2.2. Типы задач p-центров

В таблице 2 приведены публикации и типы задач p-центров.

Таблица 2. Публикации и типы задач p-центров, часть 1

Ном.	Публикация, тип задачи	Источники
1.	Обзоры	
1.1.	Survey on p-center problems (problem variants, methods)	[6, 36]
1.2.	The p-center and p-median problems (location on networks)	[2, 37]
1.3.	Survey on k-center problems (problems, approximation)	[35]
1.4.	k-center problems (problems, complexity issues)	[4]
2.	Основные типы задач:	
2.1.	Basic p-center (k-median) problems	[5, 24, 33, 38]
2.2.	Weighted p-center problems	[39, 40]
2.3.	Connected p-center problem (problem, algorithms, applications)	[32]
2.4.	Two-center problems	[41, 42, 43, 44]
2.5.	Multi-centers problem on a graph (optimum location)	[38, 45]
2.6.	Multifacility center problems	[46, 47]
2.7.	p-center problem with minimum coverage (balanced version)	[13, 48]
2.8.	The multi-service center problem	[50]
2.9.	Alternative p-center problems	[51]
2.10.	Fault tolerant p-center problems	[52]
2.11.	Hierarchical version of p-center problem	[53]
2.12.	Priority p-center problem	[54]
2.13.	Discrete p-center problems	[55, 56]
2.14.	Generalized p-center problems	[57]
3.	Вершинные задачи p-центров (vertex p-center problems):	
3.1.	Vertex p-center problem	[58, 59, 60, 61, 62]
3.2.	Robust vertex p-center model for locating urgent relief distribution centers	[18]
3.3.	The complete vertex p-center problem	[61]
3.4.	Capacitated vertex p-center problems	[63, 64, 65]
3.5.	The connected p-vertex one-center problem on graphs	[66]
3.6.	Large unconditional and conditional vertex p-center problems	[67]
3.7.	Large-scale capacitated vertex p-center problem	[68]
3.8.	Vertex restricted p-center problems on networks	[69]
3.9.	Uniform capacitated vertex p-center problem	[70]
4.	Задачи с ресурсными ограничениями (capacitated p-center problems):	
4.1.	Capacitated p-center problem	[71, 72, 73]
4.2.	Capacitated fault-tolerant p-center problem	[74]
4.3.	Heterogeneous capacitated p-center problem	[75]
4.4.	Capacitated p-center problem with failure foresight	[76]
4.5.	The fault-tolerant capacitated p-center problem	[71]
4.6.	Capacitated vertex p-center problems	[63, 64, 65, 68]
4.7.	Uniform capacitated vertex p-center problem	[70]
5.	Асимметричные задачи:	
5.1.	Asymmetric p-center problems	[77, 78, 79, 80]
5.2.	Asymmetry weighted p-center problem	[79]
5.3.	Asymmetric p-center with minimum coverage	[48]
6.	Планарные задачи и задачи с Эвклидовой метрикой:	
6.1.	p-center in planar graphs	[3, 81, 82]
6.2.	1-center problem on the plane (with uniformly distributed demand points)	[83]
6.3.	Planar 2-center problem	[84, 85, 86, 87, 44]
6.4.	The weighted Euclidean 1-center problem	[88]
6.5.	2D Euclidean 2-center with outliers	[42]

Таблица 2. Публикации и типы задач p-центров, часть 2

Ном.	Публикация, тип задачи	Источники
7.	Задачи на деревьях:	
7.1.	Center location problems on tree graphs	[89, 90, 32, 91]
7.2.	1-center problem in tree	[92, 93]
7.3.	Continuous p-center problem on a tree	[94, 95]
7.4.	The weighted 3-center and 4-center problems in trees	[96]
7.5.	Equal capacity p-center problem on tree	[49]
7.6.	Connected p-center problem on tree	[32]
7.7.	p-centers on a weighted tree	[97, 98, 99]
7.8.	Upgrading the 1-center problem with edge length variables on a tree	[100]
7.9.	Location on tree networks: p-center and n-dispersion problems	[101]
7.10.	Maintaining center and median in dynamic trees	[102]
8.	Two-center problems:	
8.1.	Planar 2-center problems	[85, 86, 44]
8.2.	Discrete 2-center problem	[41]
8.3.	2D Euclidean 2-center with outliers	[42]
8.4.	Proximity connected two center problem	[43]
8.5.	Bichromatic two-center problem for pairs of points	[103]
9.	Задачи в условиях неопределенности:	
9.1.	The p-center problem under uncertainty	[104]
9.2.	Minimax models for capacitated p-center problem in uncertain environment	[105]
9.3.	Computing p-centers of uncertain points on a real line	[106]
9.4.	One-dimensional p-center on uncertain data	[107]
9.5.	p-center problems under uncertainty (i.e., fuzzy estimates, etc.)	[18]
10.	Динамические задачи:	
10.1.	Dynamic p-center (p-median) problems (locating centers in a dynamically changing network)	[108]
10.2.	The multi-period p-center problem with time-dependent travel times	[109]
10.3.	p-center problem on dynamic graphs	[110]
10.4.	Dynamically second preferred p-center problem	[111]
10.5.	Means of time series	[112]
10.6.	New p-means type smooth subspace clustering for time series data	[113]
10.7.	The p-next center problem (pNCP)	[114, 115, 116, 117]
10.8.	The discrete p-center location problem with upgrading	[118]
10.9.	Robust (kinetic robust) p-center problem	[119]
10.10.	Maintaining center and median in dynamic trees	[102]
11.	Инверсные задачи:	
11.1.	Inverse center location problem	[120, 121]
11.2.	Inverse center location problem on a tree	[122, 123, 124]
11.3.	Inverse 1-center location on weighted/unweighted tree	[123, 125, 126])
11.4.	Inverse 1-center location problems with edge length augmentation on trees	[127]
11.5.	Robust reverse 1-center problems on trees with interval costs	[128]
11.6.	Inverse absolute and vertex center location problem	[129]
11.7.	Inverse absolute and vertex 1-center location problems on trees	[125]
12.	Некоторые близкие задачи:	
12.1.	Matroid center problem	[130]
12.2.	Matroid and knapsack center problem	[130]
12.3.	Basic knapsack center problem	[34, 130, 131, 132]
12.4.	Outlier version of knapsack center problem	[130]
12.5.	Multi-knapsack center problem	[130]
12.6.	The most degree-central clique problem	[133]

Таблица 2. Публикации и типы задач p-центров, часть 3

Ном.	Публикация, тип задачи	Источники
13.	Специальные типы задач:	
13.1.	The aligned p-center problem	[134]
13.2.	Rectangular p-center problem	[20]
13.4.	Locating collection centers for incentive-dependent returns under a pick-up policy with capacitated vehicles	[135]
13.5.	Fair p-center problem with outliers on massive data (machine learning, distributed algorithm)	[136]
13.6.	The p-neighbor k-center problem	[137]
13.7.	α-neighbor p-center problem ($p - center^{\alpha}$)	[138]
13.8.	q-coverage p-center problems	[13]
13.9.	Colorful p-center problem	[139, 140]
13.10.	α-neighbor p-center problem	[141, 142]
13.11.	Minimum edge-dilation p-center problem (for edge-weighted undirected and connected graph)	[143, 144]
13.12.	Compact MILP formulations for the p-center problem	[145]
13.13.	Star p-hub center problem (with bounded path lengths)	[22]
13.14.	The 2-mixed-center color spanning problem	[146]
13.15.	Bichromatic two-center problem for pairs of points	[103]
13.16.	The most vital elements for 1-center and 1-median location problems	[147]
13.17.	Kinetic robust p-center problem	[119]
13.18.	The stratified p-center problem	[148]
13.19.	Multi-objective (multicriteria) p-center problems	[37]
13.20.	Special generalized p-center problem (distinguishing doubling and highway dimension)	[149]
13.21.	Steiner centers in graphs	[150]
14.	Задачи размещения/назначения:	
14.1.	p-center facility location problems	[12, 13, 39, 151, 152]
14.2.	Multi-center location problems on networks	[38, 45, 69]
14.3.	Location on tree networks: p-center and n-dispersion problems	[101]
14.4.	The discrete p-center location problem with upgrading	[118]
14.5.	Locating centers in a dynamically changing network)	[108]
15.	Близкие задачи кластеризации с k центрами:	
15.1.	k-center clustering problems (continuous k-center clustering, discrete k-center clustering, parallel k-center clustering)	[153, 8]
15.2.	Fair k-center clustering (for data summarization)	[29, 205]
15.3.	Fair colorful k-center clustering	[154]
15.4.	Fair k-center clustering in MapReduce and streaming settings	[155]
15.5.	k-center clustering with outliers on massive data	[156]
15.6.	Streaming algorithms for k-center clustering with outliers	[157]
15.7.	Scalable approximation algorithm for k-center fair clustering	[158]
15.8.	k-center clustering (with outliers) in MapReduce and streaming	[159]
15.9.	Fully dynamic k-center clustering problem	[160, 161]
15.10.	Fully dynamic consistent k-center clustering problem	[162]
15.11.	k-center clustering with outliers and coreset construction	[163]
15.12.	Dynamic consistent k-center clustering with optimal recourse (survey)	[164]
15.13.	k-center clustering with outliers	[165]
15.14.	k-center clustering in distributed models	[166]
15.15.	Red-blue k-center clustering with distance constraints	[167]
15.16.	Joint cluster analysis of attribute data and relationship data (connected k-center problem, algorithms, applications)	[32]

2.3. Методы решения задач p-центров

В таблице 3 приведены публикации и типы методов решения задач p-центров.

Таблица 3. Публикации, типы методов, часть 1

Ном.	Публикация, тип метода	Источники
1.	Эвристики:	
1.1.	Heuristic for the p-center problem in graphs	[3, 34, 168]
1.2.	Heuristic and optimal algorithms for p-center problems	[169]
1.3.	Heuristic algorithm for k-center problem with vertex weight	[40]
1.4.	Lexicographic local search and the p-center problem	[170]
1.5.	Bee colony optimization for the p-center problem	[171]
1.6.	VNS method for the conditional p-next center problem	[172]
1.7.	GRASP and VNS for solving the p-next center problem	[116]
1.8.	Weighted-based tabu search for p-next center problem (pNCP)	[117]
1.9.	Vertex weighting-based double-tabu search algorithm for classical p-center problem	[173]
1.10.	Greedy strategy for k-center clustering with outliers	[163]
1.11.	Two clustering-based heuristics for the p-center problem ($O(n^3)$ time algorithms)	[174]
1.12.	Constructive heuristic for the uniform capacitated vertex p-center problem	[70]
1.13.	Iterated local search for the minimum edge-dilation p-center problem	[143]
1.14.	Artificial bee colony algorithm for the minimum edge-dilation p-center problem	[144]
1.15.	Heuristic algorithm for p-center problem with vertex weight	[40]
1.16.	Large scale local search heuristic for the capacitated vertex p-center problem	[68]
2.	Мета-эвристики, составные схемы решения:	
2.1.	Hybrid meta-heuristic with VNS and exact methods for large vertex p-center problems	[67]
2.2.	Combination of two methods: (i) minimum dominating set based algorithm, (ii) greedy algorithm	[175]
2.3.	Meta-heuristic local based approach for quadratic p-median problem	[176]
2.5.	Survey of metaheuristic approaches for p-median problem	[177]
2.6.	VNS meta-heuristic for p-median based formulations with backbone facility locations (mixed integer linear programming models)	[178]
2.7.	Iterated greedy local search with VNS (for the capacitated vertex p-center problem)	[65]
2.8.	GRASP with strategic oscillation for the α-neighbor p-center problem	[142]
2.9.	Combination of tabu search and VNS for p-center problem	[179]
3.	Приближенные алгоритмы:	
3.1.	Approximation algorithm for kinetic robust k-center problem	[119]
3.2.	3-approximation algorithm for the the vertex k-center problem	[180]
3.3.	Approximation algorithm for the edge-dilation k-center problem	[181]
3.4.	Approximation algorithms for the vertex k-center problem (survey and experimental evaluation, polynomial time heuristics)	[60]
3.5.	Approximation algorithms for asymmetric k-center problem with minimum coverage	[48]
3.6.	Improved approximation algorithm for the distributed k-center problem	[182]
3.7.	Approximating k-center in planar graphs	[81]
3.8.	$O(log^*k)$ approximation algorithms for asymmetric k-center problem	[77, 80]
3.9.	Approximation techniques for k-center with covering constraints	[139]
3.10.	Approximation algorithms for capacitated fault-tolerant k-center problem	[74]
3.11.	Greedy approximation for group centrality maximization in large-scale graph	[183]

Таблица 3. Публикации, типы методов, часть 2

Ном.	Публикация, тип метода	Источники
4.	Эволюционные методы:	
4.1.	Evolutionary approach for the p-next center problem	[115]
4.2.	GA for the minimum edge-dilation k-center problem	[184]
4.3.	Memetic GA for vertex p-center problem	[62]
5.	Методы на основе декомпозиции (т.е., разбиения задачи на подзадачи):	
5.1.	Tree decomposition algorithm for network p-center location problems	[151]
5.2.	Clustering-based exact algorithm for the p-center problem	[185]
5.3.	Decomposition approach for the p-median problem on disconnected graphs (decomposition of problem into small size subproblems)	[186]
5.4.	Efficient Benders decomposition of the p-median problem (two-phase Benders decomposition)	[187]
5.5.	Bender's decomposition for quadratic p-median problem	[176]
6.	Точные и переборные методы:	
6.1.	Exact algorithm for the capacitated vertex k-center problem	[64]
6.2.	Scalable exact algorithm for vertex p-center problem	[59]
6.3.	Branch-and-bound for k-center problem	[188]
6.4.	Scaleable projection-based branch-and-cut algorithm for the p-center problem	[189]
6.5.	Mixed breadth-depth first strategy for the branch-and-bound tree of Euclidean k-center problems	[190]
7.	Полиномиальные методы:	
7.1.	Linear time algorithm for the weighted k-center problem on trees for fixed k	[97]
7.2.	$O(n\ log\ n)$-time algorithm for the k-center problem on tree	[191]
7.3.	$O((n\ log\ p)^2)$ algorithm for continuous p-center problem on a tree	[94]
7.4.	Near-linear algorithm (for planar 2-center problem)	[85]
7.5.	Simple $O(n\ log^2\ n)$ algorithms for planar 2-center problem	[86]
7.6.	$O(n^2\ log\ n)$ algorithm for the proximity connected two center problem	[43]
7.7.	$O(n\ log\ n)$ randomizing algorithm for the weighted Euclidean 1-center problem	[88]
7.8.	$O(n\ log\ n)$-time algorithm, for planar two-center problem	[84]
7.9.	Polynomial exact algorithm for connected p-center problem on tree (dynamic programming, runtime complexity equals $O(n^2\ log^2\ ))$	[32]
7.10.	$O(n\ log\ n)$ algorithm for the weighted 3-center and 4-center problems in trees	[96]
7.11.	Linear time and space algorithm for p-center problem in tree (based on tree partitioning)	[14, 90]
7.12.	Polynomially bounded algorithms for locating p-centers on a tree	[192]
7.13.	Unified polynomial dynamic programming algorithms for p-center variants in a 2D Pareto front	[193]
7.14.	Polynomial algorithm for the equal capacity p-center problem on tree	[49]
7.15.	$O(log^*k)$ approximation algorithms for asymmetric k-center problem	[77, 80]
8.	Приближенные полиномиальные схемы с ограниченной относительной погрешностью (polynomial/fully polynomial approximate schemes - PTAS, FPTAS):	
8.1.	Polynomial time $(1+\epsilon)$-approximation scheme (PTAS) for Euclidean k-center clustering problem	[194]
8.2.	Approximation schemes (PTAS) for special k-center problem in graphs	[195]
8.3.	Polynomial-time approximation schemes for k-center problem	[196]
9.	Динамические методы:	
9.1.	Dynamic algorithms for k-center problem on graphs	[110]
9.2.	Dynamic consistent algorithms for k-center clustering problem	[162]
9.3.	Optimal fully dynamic k-center clustering	[197]
9.4.	Fully dynamic k-center clustering	[160]

Таблица 3. Публикации, типы методов, часть 3

Ном.	Публикация, тип метода	Источники
10.	Специальные методы:	
10.1.	Tight approximation algorithms for the k-center and matroid center problems with outliers	[198]
10.2.	Distributed algorithms for finding centers and medians in networks	[199]
10.3.	Efficient solution approaches for the bi-criteria p-hub median and dispersion problem	[200]
10.4.	New relaxation-based algorithms for continuous and discrete p-center problems	[10]
10.5.	Centrality of trees for capacitated k-center	[201]
10.6.	Efficient algorithms for the one-dimensional k-center problem	[202]
10.7.	Algorithms for finding p-centers on a weighted tree	[99]
10.8.	Fixed-parameter algorithms for (k,r)-center in planar graphs and map graphs	[203]
10.9.	Cuckoo search algorithm with Q-learning and genetic operation for logistics distribution center location	[15]
10.10.	Graph attention-based policy gradient method with an adaptive embedding strategy for p-center problems	[204]
10.11.	Parallel mayfly algorithm for the α-neighbor p-center problem	[141]
10.12.	Composite method: (i) GRASP, (ii) Tabu search, (iii) strategic oscillation post-processing (for the α-neighbor p-center problem)	[142]
10.13.	Sequential algorithms for distributed fair k-center clustering	[205]
10.14.	Solving the k-center problem efficiently with dominating set algorithm	[206]
10.15.	Double bound method for solving the p-center location problem	[69]
10.16.	Harmony search for conditional and unconditional p-center problem	[207]
10.17.	Structure-driven randomized algorithm for metric discrete k-center problem.	[56]
11.	Специальные составные схемы и гибридные методы:	
11.1.	Effective approaches to solve p-center problem via set covering and SAT	[208]
11.2.	Hybrid meta-heuristic with VNS and exact methods for large unconditional and conditional vertex p-center problems	[67]

Следует отметить, что задача p-центров на древовидных структурах близка к задачи разбиения дерева [90]:

Разбить дерево с весами вершин на основе удаления p ребер так, чтобы минимизировать максимальный вес компонентов (частей дерева) или максимизировать минимальный вес компонента.

Техника решения данного типа задач может быть использована для полиномиального (линейного) алгоритма решения задачи p-центров на дереве.

2.4. Некоторые модели комбинаторной оптимизации

Дискретная задача p-центров на сети

Модель целочисленного программирования имеет следующий вид [2, 118, 189, 209]. Используются обозначения:

(1) множество узлов - пользователей/клиентов (demand nodes) $A = \{i\}$;

(2) множество центров-кандидатов $B = \{j\}$;

(3) предварительно заданное число искомых центров p,

(4) неотрицательная стоимость (вес, "путь"обслуживания) (удовлетворяет неравенству треугольника) для подключения узлов $i \in A$ к центру $j \in B$ (эта стоимость может соответствовать расстоянию/близости, времени в пути и т.п.);

(5) Булевы переменные:

(5.1) переменная y_j $(j \in B)$ равна 1 если узел j выбран как центр;

(5.2) переменная x_{ij} $(i \in A), j \in B)$ равна 1 если узел $i \in A$ подключается к центру $j \in B$;

(6) целевая функция задачи Z заключается в выборе p центров с целью минимизации максимальной стоимости подключения узлов к выбранным центрам.

Комбинаторная оптимизационная модель имеет вид:

$$\min \ Z \tag{1}$$

$$s.t. \ \sum_{j \in B} c_{ij} x_{ij} \le Z, \quad \forall i \in A, \tag{2}$$

$$\sum_{j \in B} x_{ij} = 1, \quad \forall i \in A, \tag{3}$$

$$x_{ij} \le y_j \ \ \forall i \in A, \ \forall j \in B, \tag{4}$$

$$\sum_{j \in B} y_i = p, \tag{5}$$

$$x_{ij}, y_j \in \{0, 1\}, \ i \in A, \ j \in B. \tag{6}$$

Стоимость (расстояние, вес)) c_{ij} можно рассматривать в виде вектора $\overline{c}_{ij} = \{c^1_{ij}, ..., c^\mu_{ij}, ..., c^\lambda_{ij}\}$. В результате получается многокритериальная модель:

$$\min \overline{Z} = (\min \ Z^1, ..., \min \ Z^\mu, \min \ Z^\lambda), \tag{7}$$

$$s.t. \ \sum_{j \in B} c^1_{ij} x_{ij} \le Z^1, ..., \sum_{j \in B} c^\mu_{ij} x_{ij} \le Z^\mu, ..., \sum_{j \in B} c^\lambda_{ij} x_{ij} \le Z^\lambda, \ \forall i \in A, \quad (9)$$

$$\sum_{j \in B} x_{ij} = 1, \quad \forall i \in A, \tag{9}$$

$$x_{ij} \le y_j \ \ \forall i \in A, \ \forall j \in B, \tag{10}$$

$$\sum_{j \in B} y_i = p, \tag{11}$$

$$x_{ij}, y_j \in \{0, 1\}, \ i \in A, \ j \in B. \tag{12}$$

Следует отметить, что в многокритериальных моделях целесообразно искать решения в виде Парето-эффективных точек.

Задача максимизации групповой близости

Задача максимизации групповой близости (group closeness maximization problem) часто рассматривается в качестве критической при анализе графов/сетей [210, 211]. Задача имеет вид [211, 212, 213]:

Найти "центральную"группу вершин (узлов) в исходном графе (сети).

В таблице 4 приведены некоторые исследования/публикации по этой задаче. Данная задача важна при поиске наиболее "важных"вершин (узлов) в исходных графах (сетях), например, концепций в сетях информационных объектов [211, 214].

Таблица 4. Исследования по задаче максимизации групповой близости

Ном.	Исследование	Источники
1.	Общие исследования:	
1.1.	Вопросы центральности в сетях	[211]
1.2.	Математические модели для групповых структур	[210]
1.3.	Групповая центральность (community-based) в сложных сетях	[214]
2.	Задачи и исследования :	
2.1.	Задачи максимизации групповой близости	[213, 215]
2.2.	Центральность по близости для "важных"k узлов в сетях	[216, 217]
2.3.	Идентификация "важных"k узлов в сетях большой размерности	[218]
3.	Методы решения:	
3.1.	Метод эффективной идентификации	[219]
3.2.	Метод локального поиска (в больших графах)	[212]
3.3.	Точные алгоритмы для выделения "важных"k узлов в сетях	[215]
3.4.	Эвристики (обзор)	[212]
3.5.	Метод идентификации наиболее влиятельных (influential) узлов в сложных сетях на основе групп узлов (communities)	[214]
3.6.	Быстрый алгоритм для выделения "важных"k узлов в сетях	[220]

Базовая модель задачи максимизации групповой близости описана в [215]. Имеется исходный граф $G = (A, E)$: A - множество вершин ($|A| = n$), E - множество ребер ($|E| = m$). Рассматриваются типы расстояний/близости:

(1) кратчайший путь между вершинами графа $\forall a, a' \in A$.

(2) расстояние между вершиной $a \in A$ и подмножеством вершин $S \subseteq A$:

$d(a, S) = \min_{s \in S} d(a, s)$.

(3) индекс центральности по близости для вершины $a \in A$:

$c(a) = (n-1)/\sum_{a' \neq a} \ d(a, a') \ \ (a, a' \in A)$.

(4) близость множества S (групповая близость): $c(S) = (n - |S|)/\sum_{a \overline{\in} S} \ d(S, a)$

Задача имеет вид:

Найти подмножество $S^* \subseteq A$ заданного размера (мощности) k ($k < n$) с максимальной групповой близостью:

$$S^* = \arg\max_{S \subseteq A} \{c(S) : |S| = k\}.$$

Рис. 4 иллюстрирует задачу.

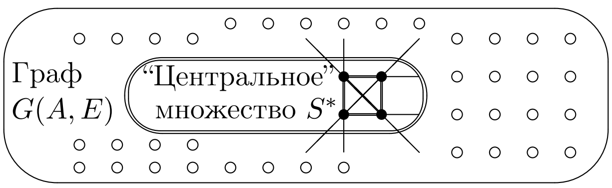


Рис. 4. Иллюстрация групповой близости

3. ЗАДАЧИ P-МЕДИАН

3.1. Описание

Базовая задача медианы имеет вид [19]:

Найти "абсолютную"медиану как точку в сети такую, что сумма взвешенных расстояний между этой точкой и другими узлами была минимальной.

Далее можно рассматривать поиск p медиан [2, 4, 23]:

Найти p медиан в сети так, чтобы сумма взвешенных расстояний между узлами сети и их ближайшими медианами были минимальной.

Таблица 5 содержит некоторые основные публикации по исследованиям и типам задач p-медиан.

Table 5. Задачи p-медиан

Ном.	Исследование/задача	Источники
1.	Обзоры:	
1.1.	Survey on p-center and p-median problems	[2, 37]
1.2.	Recent surveys on p-median problems	[177, 221]
2.	Основные типы задач p-медиан:	
2.1.	The p-median problems	[2, 4, 19, 24]
2.2.	Large p-median problem	[222, 223, 224, 225, 226]
2.3.	The planar p-median problem	[227]
2.4.	Capacitated p-median problem	[228, 229, 230, 231, 232, 233]
2.5.	Generalizations of p-median problems	[234, 235]
2.6.	Dynamic p-median problem (with mobile facilities)	[236]
2.7.	Inverse p-median problems	[237]
3.	Задачи p-медиан на деревьях:	
3.1.	p-median problem on tree graphs	[238, 239]
3.2.	2-medians problems in trees	[240, 241, 242]
3.3.	Robust inverse median problems on trees with interval costs	[243]
3.4.	k-median on a directed tree	[244]
3.5.	Localizing 2-medians on probabilistic and deterministic tree networks	[245]
3.6.	Capacitated balanced 2-median problem on tree network	[246]
4.	Специальные типы задач p-медиан:	
4.1.	The Euclidean p-median problem with uniform weights	[247]
4.2.	The pos/neg weighted p-median problem	[248, 249]
4.3.	The p-median problem under uncertainty	[250]
4.4.	Multiple p-median problem	[251]
4.5.	Neural model for the p-median problem	[252]
4.6.	The reliable p-median problem with at-facility service	[253, 254]
4.7.	Discrete ordered median problems	[255, 256, 257, 258]
4.8.	Ordered median hub location problems	[259, 260]
4.9.	Diversity-aware k-median: clustering with fair center representation	[261]
4.10.	Online median problem (metric uncapacitated k-median problem)	[262]
4.11.	p-median problem with backbone facility locations	[178]
4.12.	Traveling k-median problem (optimal network coverage)	[263]
4.13.	Vector assignment ordered median problem	[264]
5.	Специальные исследования:	
5.1.	Comparison of p-median and maximal coverage location models	[265]
5.2.	From Steiner centers to Steiner medians	[266]
5.3.	Knapsack median (generalization of k-median problem)	[267]
5.4.	The p-median structure as a linear model for location/allocation analysis	[268]

Основные методы решения задачи p-медиан указаны в таблице 6.

Таблица 6. Основные методы решения задачи p-медиан, часть 1

Ном.	Метод, исследование	Источники
1.	Обзоры:	
1.1.	Solution methods for the p-median problem: an annotated bibliography	[269]
1.2.	Survey of metaheuristic approaches for the p-median problem	[177]
1.3.	Evaluation of heuristics for p-median problem	[270]
2.	Эвристики и мета-эвристики:	
2.1.	Local search heuristics for k-median problem	[271]
2.2.	Local search heuristics for capacitated p-median problem	[228]
2.3.	Several basic heuristics for p-median problems	[2]
2.4.	Ant colony algorithm for weighted p-median problem	[249]
2.5.	Algorithms of ant system and simulated annealing for the p-median problem	[272]
2.6.	Optimization-based Lagrangian algorithm for p-median problems	[2]
2.7.	Reverse greedy algorithm for metric K-median problem	[273]
2.8.	Hybrid heuristic for the p-median problem	[274]
2.9.	Discrete PSO for p-median problem	[275]
2.10.	Aggregation heuristic for large scale p-median problem	[223]
2.11.	Efficient tabu search procedure for the p-median problem	[276]
2.12.	Heuristic algorithms for planar p-median problem	[227]
2.13.	Three heuristics for dynamic p-median problem	[236]
2.14.	Local search approximation schemes for k-median in Euclidean metrics	[277]
2.15.	Hybrid modified PSO for inverse p-median location problem (fuzzy random environment)	[278]
2.16.	General framework for local search applied to the continuous p-median problem	[279]
3.	Эволюционные методы:	
3.1.	Efficient GA for p-median problem	[280]
3.2.	GAs for the discrete ordered median problem	[258]
3.3.	Efficient heuristic method (GA) for capacitated p-median problem	[231]
3.4.	Special GA capacitated p-median problem	[230]
3.5.	Parallel GA for capacitated p-median problem	[232]
4.	Методы поиска на основе переменных окрестностей (VNS):	
4.1.	VNS for p-median problem	[281]
4.2.	VNS for p-median problem with weights	[248]
4.3.	VNS meta-heuristic for p-median based formulations with backbone facility location (mixed integer linear programming models)	[178]
5.	Полиномиальные методы:	
5.1.	Computing 2-median on tree networks in $O(n\ log\ n)$ time	[238]
5.2.	$O(p\ n^2)$ algorithm for p-median on tree graphs	[239]
5.3.	$O(p\ n^2)$ algorithm for the p-median and related problems on tree graphs	[239]
5.4.	Efficient algorithms for two generalized 2-median problems and the group median problem on trees	[242]
6.	Переборные подходы:	
6.1.	Branch-and-price approach to p-median location problem	[282]
6.2.	Branch-and-price algorithm for the capacitated p-median problem	[229]
6.3.	Branch-and-bound algorithm for the capacitated p-median problem	[229]
6.4.	Branch-and-price algorithm for dynamic p-median problem	[236]
7.	Методы на основе динамического программирования:	
7.1.	Branch decomposition based dynamic programming algorithm for p-median problem	[283]
7.2.	Dynamic programming heuristic for p-median problem	[284]

Таблица 6. Основные методы решения задачи p-медиан, часть 2

Ном.	Метод, исследование	Источники
8.	Специальные методы решения:	
8.1.	Fast and robust techniques for the Euclidean p-median problem with uniform weights	[247]
8.2.	Solving the p-median problem with semi-Lagrangian relaxation	[285]
8.3.	Tree search algorithm for the p-median problem	[286]
8.4.	Efficent neural network algorithm for the p-median problem	[287]
8.5.	Column generation approach (iterative decomposition method) to capacitated p-median problems	[233]
8.6.	Benders decomposition (Branch-and-Benders-cut) for discrete ordered median problem	[256]

3.2. Модели комбинаторной оптимизации

Дискретная задача p-медиан

Дискретная задача p-медиан заключается в размещении p единиц оборудования (медиан, центров) с целью минимизации общего взвешенного расстояния между оборудованием и пользователями (клиентами). Эта задача является расширением известной задачи Вебера [288,289,290] за счет использования набора оборудования. Комбинаторная оптимизационная модель задачи имеет вид [287, 291]:

$$\min \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} x_{ij} \tag{13}$$

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i = \overline{1,n} \tag{14}$$

$$\sum_{j=1}^{n} x_{jj} = p \tag{15}$$

$$x_{ij} \leq x_{jj} \quad i = \overline{1,n},\ j = \overline{1,n}, \tag{16}$$

где

n - рассматриваемое множество исходных точек-пользователей (demand points),

p - число оборудования (или медиан, центров),

d_{ij} - расстояние (стоимость, вес) между пользователями (клиентами) i и оборудованием j,

Булева переменная $x_{ij} = 1$ если i связан с оборудованием j (иначе $x_{ij} = 0$).

В случае использования векторной оценки расстояния (веса, близости) d_{ij} трансформируется в вектор: $\overline{d}_{ij} = \{d^1_{ij}, ..., d^{\mu}_{ij}, ..., d^{\lambda}_{ij}\}$. В результате получается векторная целевая функция:

$$\min \left(\sum_{i=1}^{n} \sum_{j=1}^{n} d^1_{ij} x_{ij}, ..., \sum_{i=1}^{n} \sum_{j=1}^{n} d^{\mu}_{ij} x_{ij}, ..., \sum_{i=1}^{n} \sum_{j=1}^{n} d^{\lambda}_{ij} x_{ij} \right) \tag{17}$$

Числовой иллюстративный пример для задачи p-медиан приведен на Рис. 5:

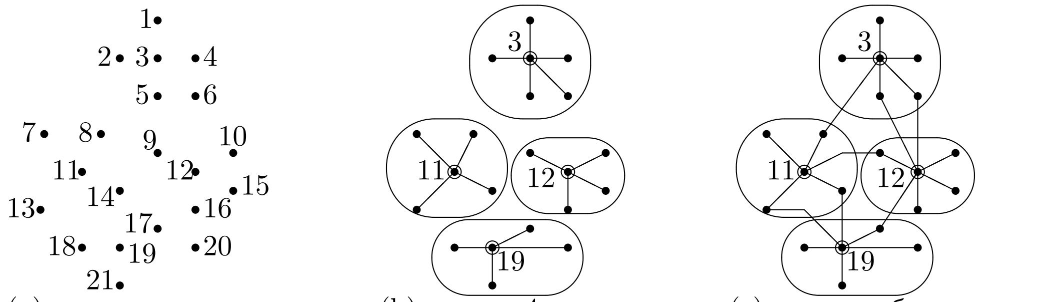
(1) исходные точки (Рис. 5a):

$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21\}$.

(2) задача 4-медиан (Рис. 5b):

(2.1) множество из 4-х медиан: $\{3, 11, 12, 19\}$;

(2.2) подключение точек к медианам представлено в таблице 7.



(a) исходные элементы (b) задача 4-медиан (c) мульти-обслуживание

Рис. 5. Иллюстрация для задач 4-медиан

Таблица 7. Присоединение точек в задаче 4-медиан

Ном.	Медиана (пункт обслуживания, оборудование)	Присоединенные точки (пользователи)
1.	3	1,2,4,5,6
2.	11	7,8,13,14
3.	12	9,10,15,16
4.	19	17,18,20,21

Задача p-медиан с ресурсным ограничением

Задача p-медиан с ресурсным ограничением (capacitated p-median problem) имеет много приложений $[228, 229, 230, 231]$. Используются следующие компоненты:

(1) исходное множество точек (узлов) $A = \{1, ..., i, ..., n\}$;

(2) подмножество потенциальных мест размещения медиан (кандидатов) $B = \{1, ..., j, ..., m\}$ $(B \subseteq A)$;

(3) целочисленные коэффициенты w_i соответствуют ресурсу, который требуется пользователю $(\forall i \in A)$;

(4) целочисленный параметр q_j соответствуют ресурсу оборудования, размещенного в центре j $(\forall j \in B)$;

(5) p единиц оборудования должны быть размещены в p различных местах;

(6) целочисленный параметр d_{ij} характеризует "расстояние"между двумя точками i и j $(\forall i, j \in A)$. Также используется дополнительное предположение: $d_{ij} \geq 0 \ \ \forall i, j \in A$.

Задача имеет вид:

Каждый пользователь (клиент) $i \in A$ должен быть подключен к одному оборудованию (медиане, центру) $j \in B$ $(B \subseteq A$ $)$ так, чтобы: (а) каждое оборудование имело достаточно ресурса для обслуживания подключенных пользователей, (б) общая стоимость обслуживания (т.е., суммарное "расстояние"от пользователей до соответствующего оборудования) была минимальной.

Получается следующая модель комбинаторной оптимизации:

$$\min \sum_{i \in A} \sum_{j \in B} \ d_{ij} x_{ij} \qquad (18)$$

$$s.t. \ \sum_{j \in B} \ x_{ij} = 1, \quad \forall i \in A, \qquad (19)$$

$$\sum_{i \in A} \ w_i x_{ij} \leq q_j y_j \ \ \forall j \in B \qquad (20)$$

$$\sum_{j \in B} \ y_j = p \qquad (21)$$

$$x_{ij} \in \{0,1\} \ \forall i \in A, \ \forall j \in B \qquad (22)$$

$$y_j \in \{0,1\} \ \ \forall j \in B \qquad (23)$$

Здесь используются следующие переменные:

(a) Булева переменная $\{x_{ij}\}$: $x_{ij} = 1$ если пользователь $i \in A$ подключен к оборудованию $j \in B$;

(b) Булева переменная $\{y\}$: $y_j = 1$ если оборудование размещается на позиции $j \in B$.

Целевая функция направлена на минимизацию суммарного "расстояния"между каждым пользователем и соответствующим оборудованием. Очевидно, что пользователи, подключенные к одному оборудованию, образуют группу (кластер), где позиция оборудования играет роль центра.

В многокритериальной постановке "расстояние"d_{ij} трансформируется в вектор
$\overline{d}_{ij} = \{d^1_{ij}, ..., d^{\mu}_{ij}, ..., d^{\lambda}_{ij}\}$.

В результате целевая функция приобретает вид вектора:

$$\min \ (\sum_{i \in A} \sum_{j \in B} \ d^1_{ij} x_{ij}, ..., \sum_{i \in A} \sum_{j \in B} \ d^{\mu}_{ij} x_{ij}, ..., \sum_{i \in A} \sum_{j \in B} \ d^{\lambda}_{ij} x_{ij} \ ) \qquad (24)$$

Дополнительно можно рассматривать векторную оценку потребностей пользователя
$w_i \Rightarrow \overline{w}_i = \{w^1_i, ...w^{\delta}_i, ..., w^{\Delta}_i\}$
и векторную оценку ресурса оборудования
$q_j \Rightarrow \overline{q}_j = \{q^1_j, ...q^{\delta}_j, ..., q^{\Delta}_j\}$.

В этом случае ограничение по ресурсу (20) изменяется:

$$\sum_{i \in A} \ \overline{w}_i x_{ij} \ \preceq \ \overline{q}_j y_j \quad \ \forall j \in B \qquad (25)$$

Задача p-медиан с многими подключениями

Задача p-медиан с многими подключениям (multiple p-median problem) рассмотрена в [251]. Здесь пользователь может быть подключен к нескольким позициям с оборудованием. Используются следующие компоненты задачи:

(1) множество точек (пользователей, клиентов) $A = \{1, ..., i, ..., n\}$;

(2) множество потенциальных позиций (кандидатов) для оборудования (для медиан, центров) $B = \{1, ..., j, ..., n\}$;

(3) "расстояние"d_{ij} между пользователем $i \in A$ и оборудованием $j \in B$;

(4) каждый пользователь $i \in A$ может быть присоединен к нескольким позициям с оборудованием, число подключений не превышает η;

(5) общее число позиций с оборудованием определяется параметром p (каждая точка может подключаться к числу оборудований η из общего числа p);

(6) Булевы переменные:

(6.1) x_{ij}: $x_{ij} = 1$ если точка i подключена к оборудованию j, ($x_{ij} = 0$ - иначе);

(6.2) y_j: $y_j = 1$ если оборудование размещено на позиции j, ($y_j = 0$ - иначе).

Модель комбинаторной оптимизации имеет вид [265, 221, 251]:

$$\min \sum_{j \in B} \sum_{i \in A} \ d_{ij} x_{ij} \qquad (26)$$

$$s.t. \ \ \sum_{j \in B} \ x_{ij} \geq \eta, \ \ \forall i \in A \qquad (27)$$

$$\sum_{j\in B} y_j = p \qquad (28)$$

$$x_{ij} - y_j \leq 0, \quad \forall i \in A, \ \forall j \in B \qquad (29)$$

$$y_j \in \{0,1\}, \quad \forall j \in B \qquad (30)$$

$$x_{ij} \in \{0,1\}, \quad \forall i \in A, \ \forall j \in B \qquad (31)$$

Здесь целевая функция (26) направлена на минимизацию общего (суммарного) "расстояния".

Дополнительно можно рассмотреть расширение модели:

(1) дополнительный параметр w_i ($\forall i \in A$) как вес обслуживания пользователя (стоимости потребности, как в предыдущей задаче);

(2) переход к многоритериальной постановке как в предыдущей задаче: "расстояние"(близость) d_{ij} трансформируется в вектор $\overline{d}_{ij} = \{d^1_{ij}, ..., d^\mu_{ij}, ..., d^\lambda_{ij}\}$. В результате целевая функция (26) преобразуется к виду:

$$\min \ (\sum_{i\in A}\sum_{j\in B} \ d^1_{ij}x_{ij}, ..., \sum_{i\in A}\sum_{j\in B} \ d^\mu_{ij}x_{ij}, ..., \sum_{i\in A}\sum_{j\in B} \ d^\lambda_{ij}x_{ij} \ ) \qquad (32)$$

На Рис. 5с приведен иллюстративный пример для данной задачи (с многими подключениями пользователей))

(1) множество исходных точек (элементов) (Рис. 5a):

$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21\}$.

(2) 4-элементное множество медиан (центров обслуживания, оборудования) (Рис. 5c):

$\{3, 11, 12, 19\}$;

(3) присоединение элементов к медианам указано в таблице 8.

Таблица 8. Присоединение в задаче 4-медиан (мульти-обслуживание)

Ном.	Медиана (пункт обслуживания, оборудование)	Присоединенные точки (пользователи)
1.	3	1,2,4,5,6,8
2.	11	7,8,9,13,14
3.	12	5,6,9,10,15,16,17
4.	19	13,14,17,18,20,21

4. О ПРОСТЫХ ЭВРИСТИЧЕСКИХ СХЕМАХ

Следует привести несколько очевидных эвристических схем решения.

Во-первых, предварительное задание множества позиций-кандидатов для размещения оборудования:

1. Экспертное задание (expert judgment).

2. Предварительная балансная кластеризация исходного графа для получения p сбалансированных кластеров и назначение в каждом кластере головной вершины (позиции-кандидата).

3. Построение в исходном графе доминирующего подмножетва и использование этого подмножества как набора позиций-кандидатов.

4. Генерация множества кандидатов на основе решения задачи покрытия исходного графа.

5. Комбинированные схемы построения множества позиций-кандидатов.

В-вторых, следующие простые эвристические схемы могут быть использованы:

Схема 1: на основе предварительной балансной кластеризации.

Схема 2: на основе аппроксимации исходного графа иерархической структурой (типа покрывающего дерева, леса). В общем случае в комбинаторной оптимизации часто используются дерево-декомпозируемые структуры (tree-decomposable):

деревья, параллельно-последовательные графы, интервальные графы и др.

Схема 3: на основе аппроксимации исходного графа планарной структурой.

Схема 4: на основе предварительной декомпозиции исходной задачи (например, использование метода декомпозиции Бендерса).

5. ПРИКЛАДНОЙ ПРИМЕР

Здесь рассматривается задача определения "центральных"исследовательских статей (публикаций) на примере набора статей в области современных сетей связи. Следует отметить, что анализ мер центральности в сетях цитирования приведен в [31]. В нашем случае целью является выделение наиболее перспективных (инновационных, "центральных") комплексных публикаций.

В таблице 9 приведены десять исследовательских публикаций в области современных систем связи. Таким образом имеется исходное множество элементов: $A = \{a_1, ..., a_i, ..., a_{10}\}$. В таблице 10 представлены соответствующие публикациям (т.е., элементам исходного множества) ключевые слова/ключевые информационные концепты (по 6 ключевых слов для каждого элемента $\forall a_i \in A$).

Таблица 9. Публикации в области современных систем связи

Элемент a_i	Публикация	Источник
a_1	Reconfigurable intelligent surface (principles and opportunities)	[292]
a_2	Reconfiguring wireless environments via intelligent surfaces for 6G	[293]
a_3	Comprehensive review on RIS for 6G communications: overview, deployment, control mechanism, application, challenges, and opportunities	[294]
a_4	Trajectory optimization of UAV-IRS assisted 6G THz network using deep reinforcement learning approach	[295]
a_5	Intelligent reflecting surface enhanced mobile edge computing (minimizing the maximum computational time)	[296]
a_6	Communication networks perspectives towards 6G (complex systems)	[297]
a_7	MEC-Empowered non-terrestrial network for 6G wide-area time-sensitive IoT	[298]
a_8	6G cognitive information theory	[299]
a_9	Survey on security and privacy for 6G	[300]
a_{10}	Survey on 6G networks with optical communications	[301]

Таблица 10. Публикации и соответствующие ключевые слова

Элемент	Ключевые слова
a_1	6G, RIS, intelligence - ML, optimization, wireless network, system architecture
a_2	6G, RIS, security, modulation, wireless, system reconfiguration
a_3	6G, MEC, RIS, control, IoT, challenges
a_4	6G, UAV, RIS, intelligence - ML, optimization, architecture
a_5	6G, IRS (RIS),, MEC, optimization, latency, computational time
a_6	6G, mobile network, modeling, wireless network, network architecture, perspectives
a_7	6G, IoT, MEC, architecture, non-terrestrial network, cell-free
a_8	6G, UAV, evolution, intelligence, data fusion, MEC
a_9	6G, ground-see network, space network, air network, security, architecture
a_{10}	6G, optical communications, optimization-allocation, P2P, architecture, perspectives

Таблица 11 содержит оценки "похожести"в виде общего числа ключевых слов для каждой пары элементов $\rho(a_{i_1}, a_{i_2})$ $\forall a_{i_1}, a_{i_2} \in A$. Близость ("расстояние") между элементами a_{i_1} и a_{i_2} можно вычислять по формуле $1 - \rho(a_{i_1}, a_{i_2})/6$.

Таблица 11. "Похожесть"$\rho(a_{i_1}, a_{i_2})$

a_{i_1}	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
a_1	3	2	6	5	3	2	2	2	3
a_2		2	2	2	4	3	1	3	2
a_3			2	3	3	2	2	1	1
a_4				3	2	2	3	3	3
a_5					1	2	2	1	2
a_6						2	1	3	3
a_7							2	4	2
a_8								2	1
a_9									1

Можно выделить 3 группы близких элементов (кластеры) на основе таблицы 11; группа 1: $X_1 = \{a_1, a_2, a_4, a_5\}$, группа 2: $X_2 = \{a_7, a_8, a_9\}$, группа 3: $X_3 = \{a_3, a_6, a_{10}\}$. Следующие элементы (публикации) являются "центральными"(на основе минимальной общей близости внутри группы): a_1 для группы 1, a_7 для группы 2, a_3 для группы 3.

6. ЗАКЛЮЧЕНИЕ

Данная работа представляет собой "инженерный"обзор задач p-центров и p-медиан, включая типы задач, методы решения, иллюстрации. Предложена новая прикладная задача выделения "центральных"публикаций в некоторой проблемной области.

В качестве перспективных направлений исследований можно указать следующие; (1) исследование задач p-центров и p-медиан с использованием порядковых и векторных оценок параметров задач; (2) исследование задач p-центров и p-медиан в условиях неопределенностей (т.е., вероятностные параметры, параметры на основе размытых множеств); (3) исследование динамических постановок задач; (4) исследование различных много-стадийных стратегий/процедур решения; (5) исследование различных приложений; (6) проектирование специальных систем поддержки решения задач p-центров и p-медиан и использование их в учебном процессе.

Автор подтверждает, что нет конфликтов интересов.

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On p-center problem

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The paper addresses p-center and p-median problems in graphs/networks. In general, the material is based on an architectural glance to domains of p-center problems and involves the following parts: (1) a survey on p-center problems, solving approaches, some problem formulations (including multicriteria version); (2) a survey on p-median problems, solving approaches, and problem formulations (including multicriteria versions); (3) illustrative numerical examples; (4) a new applied problem for selection of 'central' papers in a group of research papers on communications.

KEYWORDS: p-center problem, p-median problem, combinatorial optimization, heuristics, application